# Time-optimal holonomic quantum computation


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## Introduction

A three-level system can be used in a $\Lambda$-type configuration in order to construct a universal set of quantum gates through the use of non-Abelian non-adiabatic geometrical phases. This setup allows for high-speed operation times, protecting the system against decoherence. Unfortunately, arbitrarily short pulses introduce a breakdown of the rotating wave approximation (RWA). We investigate the trade-off between dissipative effects and the validity of the RWA in the model.

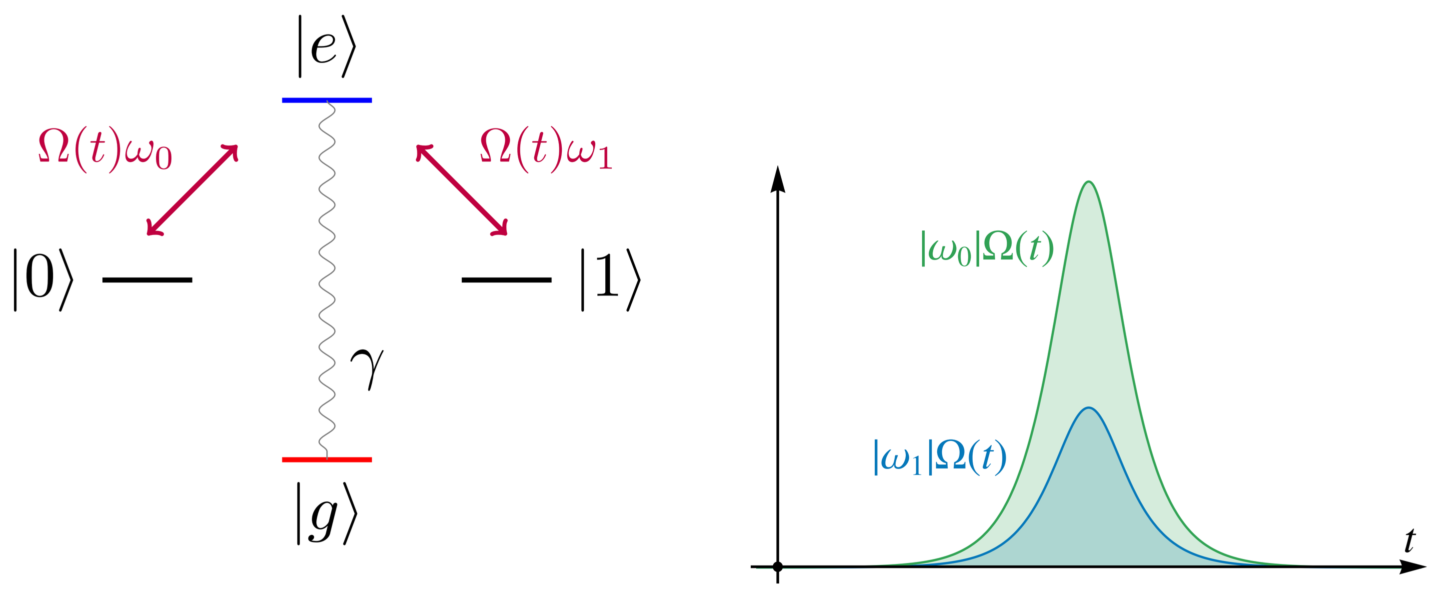
## Non-adiabatic holonomic QC

The Hamiltonian describing a $\Lambda$-type three-level system, controlled by a pulse envelope $\Omega(t)$ and with relative amplitudes $\omega_0$ and $\omega_1$ between the transition levels, is given by:[1]

$$H_I^{\mathrm{RWA}}(t) = \Omega_0(t) \left|e\right\rangle \left\langle 0\right| + \Omega_1(t) \left|e\right\rangle \left\langle 1\right| + \mathrm{h.c},$$

with $\Omega_j(t) = \omega_j \Omega(t)$. If the pulse area $\Phi := \int_0^t \Omega(t')dt'$ satisfies $\Phi = \pi$, the gate implements the holonomy matrix $U(C_{\boldsymbol{n}}) = \boldsymbol{n}\cdot\boldsymbol{\sigma}$, with $\boldsymbol{n} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$, parametrized by $\omega_0 = \sin(\theta/2)e^{i\phi}$ and $\omega_1 = -\cos(\theta/2)$. A sequence of two pulses can be used to implement a universal single-qubit gate:

$$U(C) = U(C_{\boldsymbol{m}})U(C_{\boldsymbol{n}}) = \boldsymbol{n}\cdot\boldsymbol{m} - i\boldsymbol{\sigma}\cdot(\boldsymbol{n}\times\boldsymbol{m}).$$

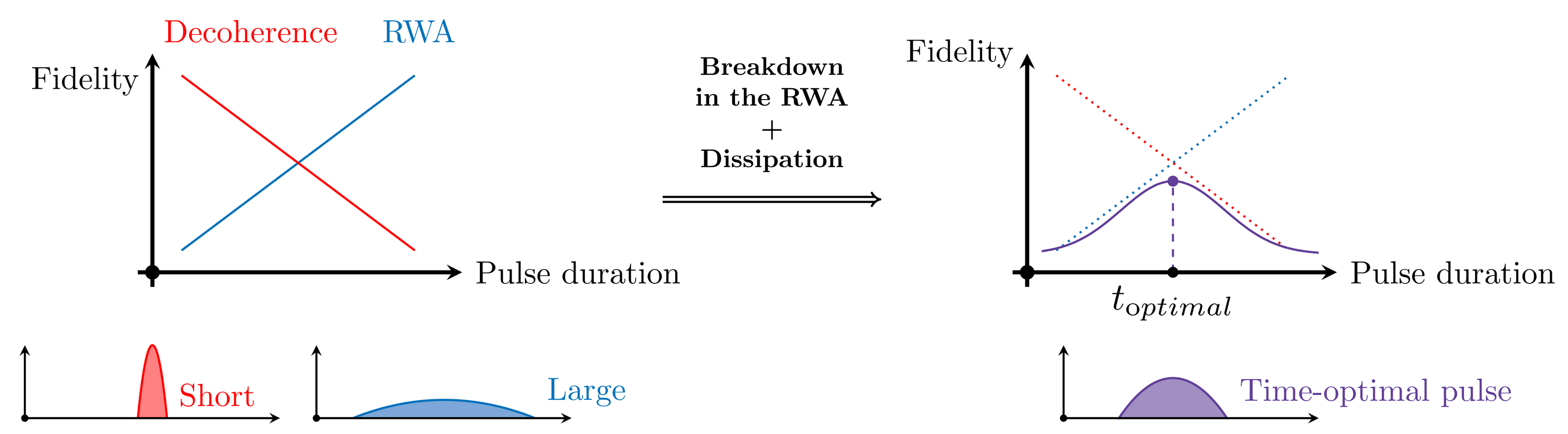



## Results

We can include counter-rotating terms $f_{e0}$ and $f_{e1}$ in order to describe the behavior of the system beyond the RWA[2]. Moreover, by rewriting the Hamiltonian in terms of its bright and dark states $|b\rangle$ and $|d\rangle$, respectively, we get:

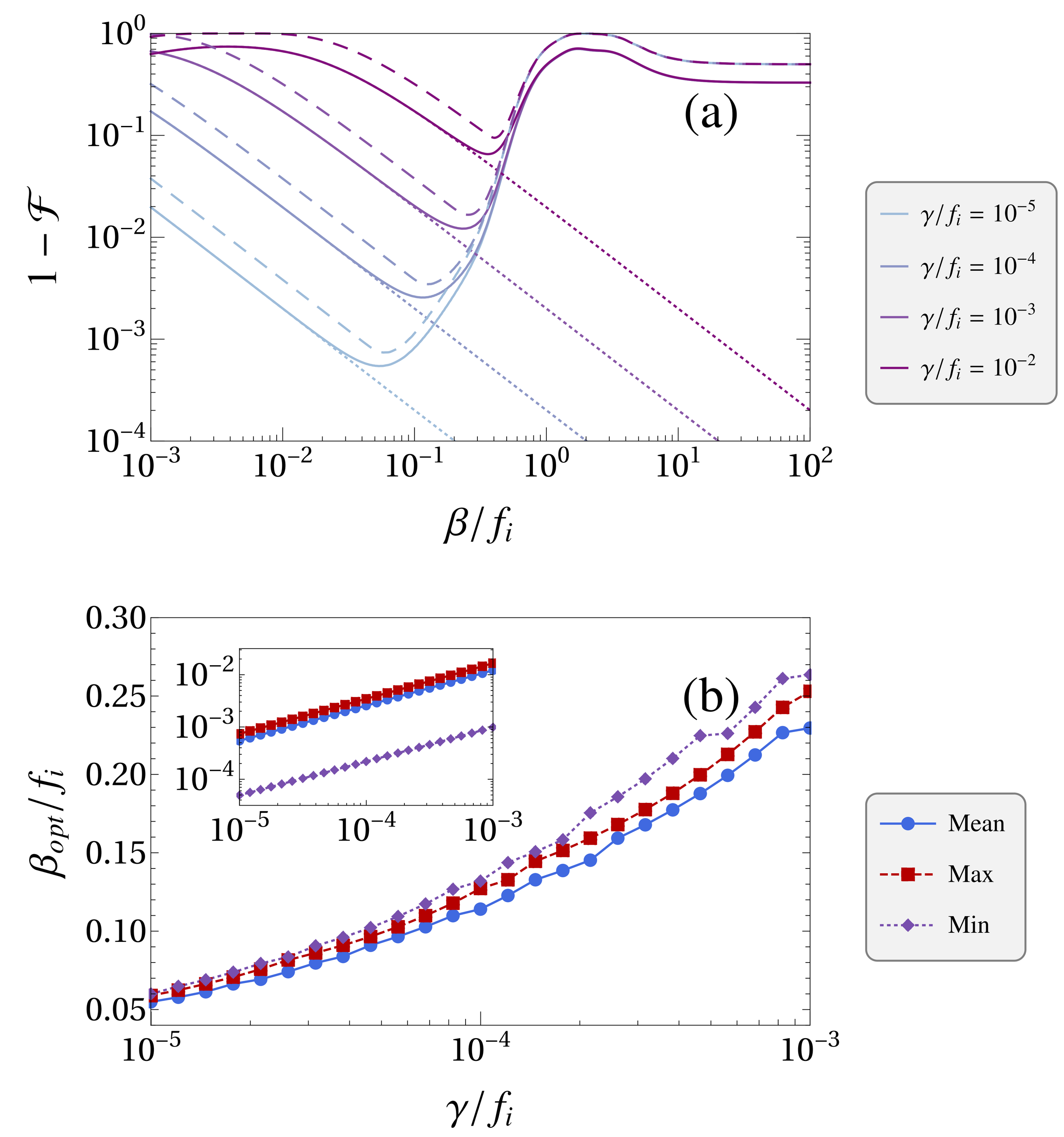
$$\begin{aligned} H_{bd}(t) = &\ \Omega(t)(1 + |\omega_0|^2 e^{-2if_{e0}t} + |\omega_1|^2 e^{-2if_{e1}t}) \left|e\right\rangle \left\langle b\right| \\ &+ \Omega(t)\omega_0\omega_1(e^{-2if_{e1}t} - e^{-2if_{e0}t}) \left|e\right\rangle \left\langle d\right| + \mathrm{h.c.} \end{aligned}$$

Here, we consider hyperbolic secant pulses of the type $\Omega(t) = \beta\,\mathrm{sech}(\beta t)$, parametrized by the inverse pulse length $\beta$. Dissipation is described through the master equation $\frac{d\rho}{dt} = i[\rho, H_I(t)] + \gamma D_\rho(\left|g\right\rangle\left\langle e\right|)$.




We observe a trade-off in the pulse duration; shorter pulses can be used to protect the system against decoherence, but they are more fragile against imperfections brought from the breakdown in the RWA. We look for an optimal regime of operation where we can find a good balance between those two effects.[3]

- ✓ **We obtain the optimal (inverse) pulse length $\beta_{\mathrm{opt}}$ as a function of the system parameters $\gamma$ and $f_{ei}$.**
- ✓ **Non-homogeneous frequencies introduce a non-ideal ideal contribution, coupling the *dark* state with the rest of the evolution.**
- ✓ **We perform simulations for the X, H, S and CZ gates. Non-homogeneous frequencies affect the protocol at different degrees.**




## References & Acknowledgements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