Bayesian estimation for collisional thermometry

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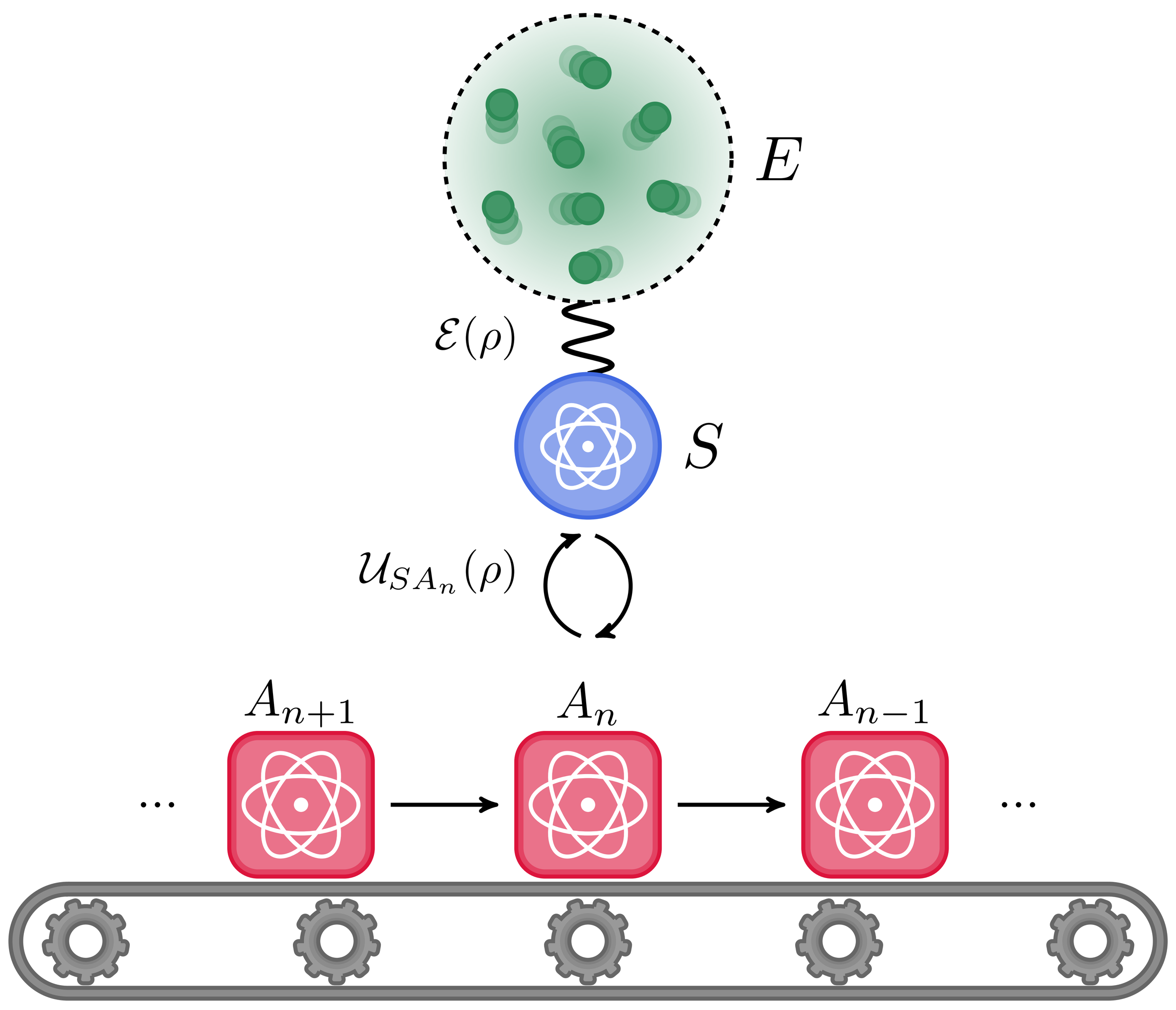
Introduction

We consider a qubit collisional model as a thermometric platform. It's been shown that the out-of-equilibrium steady state dynamics of the collisional model can be used, for instance, to enhance precision, surpassing the thermal Fisher information [1]. Here we construct a framework for analyzing collisional thermometry using Bayesian inference. In particular, we explicitly plot an estimator and compare the results with the Cramér-Rao and the Van Trees-Schützenberger bounds.



Qubit Collisional Model

The system undergoes alternating and piecewise interactions. First through a system-environment interaction for time τ_{SE}:

$$\frac{d\rho_S}{dt} = \mathcal{L}(\rho_S) = \gamma(\bar{n}+1)\mathcal{D}[\sigma_-^S] + \gamma\bar{n}\mathcal{D}[\sigma_+^S],$$

which implies $\mathcal{E}(\rho_S) = e^{\tau_{SE}\mathcal{L}}(\rho_S)$, and then through partial-swap interactions with the ancillas:

$$U_{SA_n} = \exp\left\{-i\tau_{SA}g(\sigma_+^S\sigma_-^{A_n} + \sigma_-^S\sigma_+^{A_n})\right\}$$

This results in a stroboscopic map:

$$\rho_S^n = \mathrm{tr}_{A_n}\{\mathcal{U}_{SA_n} \circ \mathcal{E}(\rho_S^{n-1} \otimes \rho_A^0)\}$$

We consider local measurements on the ancillas. The measurements are performed in the computational basis and at the steady state, which is calculated from the map above.

Bayesian Inference

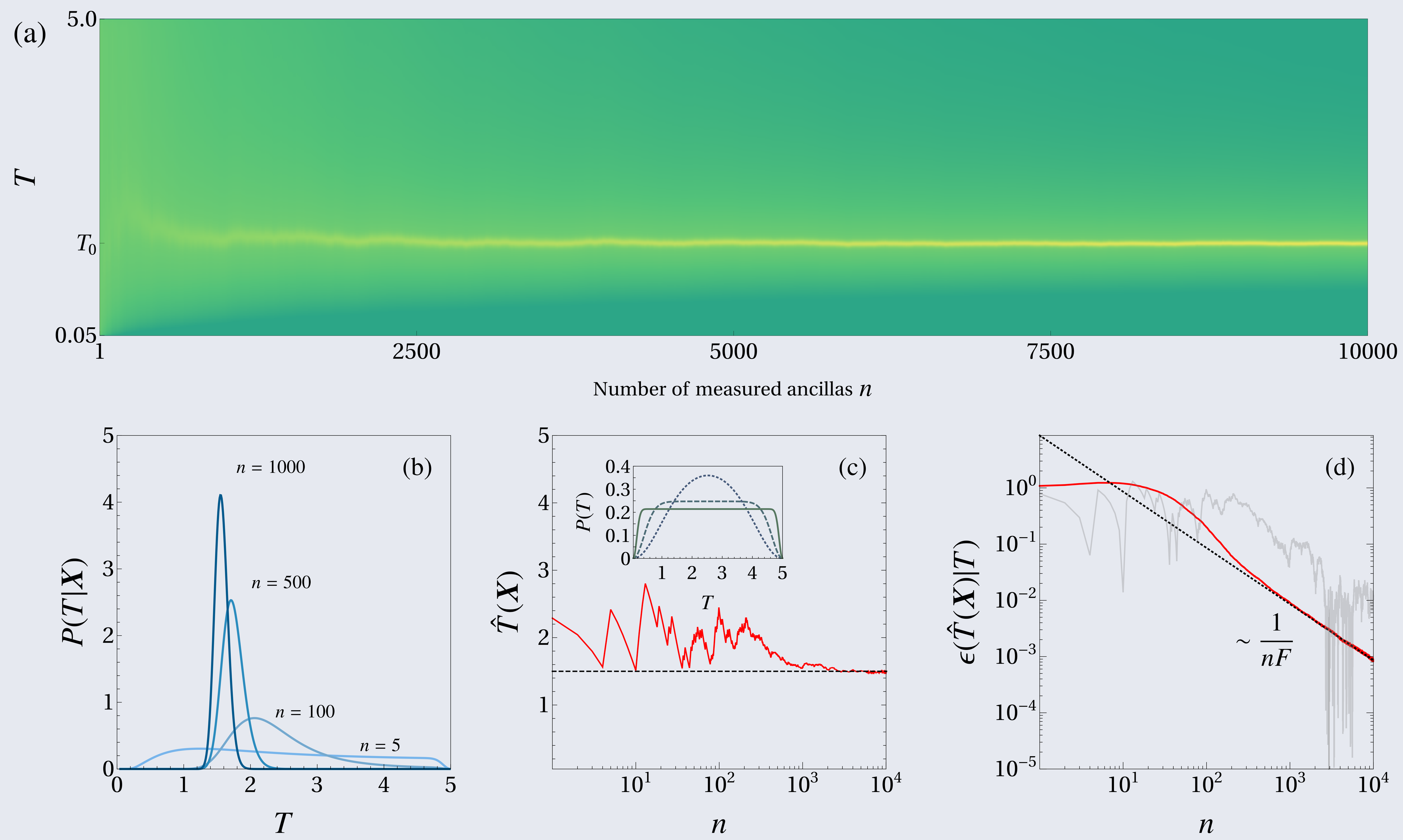
We can use Bayes theorem to construct posterior distributions $P(T|\boldsymbol{X}) \sim P(\boldsymbol{X}|T)P(T)$, which yield estimators. A natural choice of estimator is the posterior mean:

$$\hat{T}(\boldsymbol{X}) = \int T P(T|\boldsymbol{X})dT$$

The quantity above minimizes the mean-squared error $\epsilon(\hat{T}(\boldsymbol{X})|T) = \int (T-\hat{T})^2 P(\boldsymbol{X}|T)d\boldsymbol{X}$ and saturates the CRB asymptotically.

Bayesian updating

The results below show how the distribution gradually peaks around the true value of the temperature. We can also explicitly plot the estimator and the MSE as a function of n.



Additionally, the posterior distribution converges to a Gaussian, with variance proportional to the Fisher information calculated for the true temperature:

$$P(T|\boldsymbol{X}) \approx \sqrt{\frac{nF(T_0)}{2\pi}}e^{-\frac{nF_0(T-T_0)^2}{2}}, \quad (n \text{ large}).$$

We can also analyze the problem in terms of a figure of merit which is independent of the temperature. We call it the *Bayesian* error:

$$\epsilon_B(\hat{T}(\boldsymbol{X})) = \int P(T)dT \int (T-\hat{T})^2 P(\boldsymbol{X}|T)d\boldsymbol{X}$$

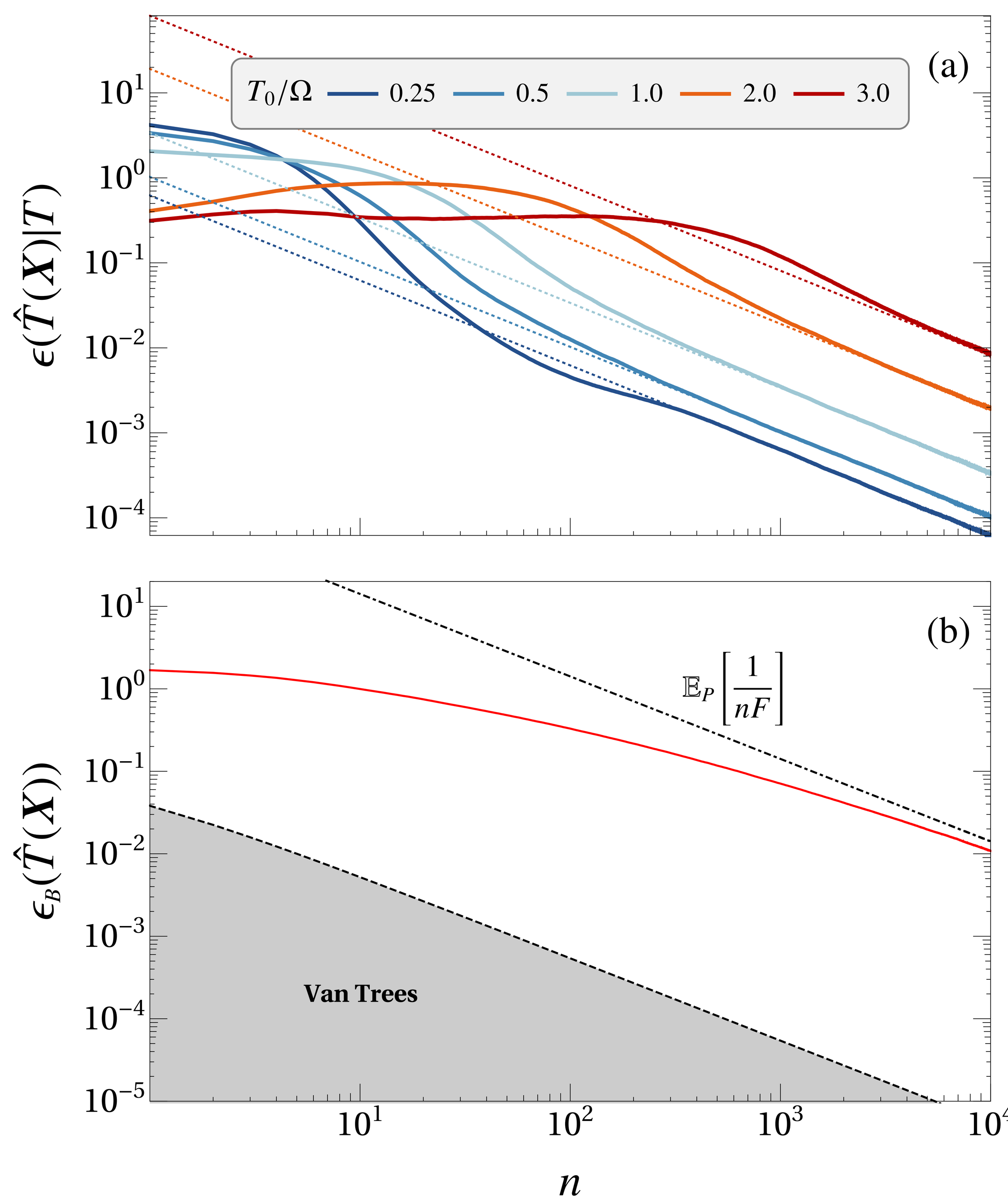
Note that the figure of merit above depends only on the estimator, the prior and the likelihood, and it's *not* conditioned neither on the outcomes nor on a particular temperature.

Van Trees-Schützenberger Inequality

This inequality establishes a bound for the Bayesian risk defined above:

$$\epsilon_B(\hat{T}(X)) \geq \frac{1}{\mathbb{E}_P[F(T)] + F_P},$$

where $\mathbb{E}_P[F(T)] = \int F(T)P(T)dT$ is the Fisher information averaged over the prior.



The figure above shows (a) the MSE for different temperatures and (b) the Bayesian risk, with the Van Trees-Schützenberger inequality in gray. Notice how the Bayesian risk converges to $\mathbb{E}_P[1/nF(T)]$, the prior-averaged CRB. This provides an asymptotic analysis which does not depend on a particular value of the (unkown) temperature. This fact makes it possible to devise strategies which are, for instance, suited to larger temperature intervals.

Acknowledgements

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References

[1] Stella Seah, Stefan Nimmrichter, Daniel Grimmer, Jader P. Santos, Valerio Scarani, and Gabriel T. Landi. Collisional Quantum Thermometry. *Physical Review Letters*, 123(18):180602, oct 2019.

[2] Gabriel O. Alves and Gabriel T. Landi. Bayesian estimation for collisional thermometry, 2021.