Probes of Full Eigenstate Thermalization in Ergodicity-Breaking Quantum Circuits

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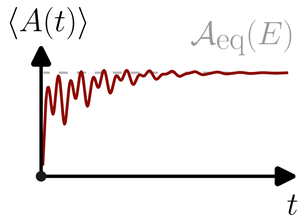
Background and motivation

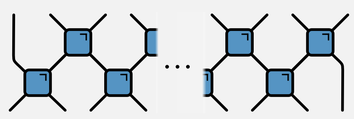
How do isolated quantum systems thermalize?
How does equilibrium statistical mechanics emerge from a non-equilibrium situation?



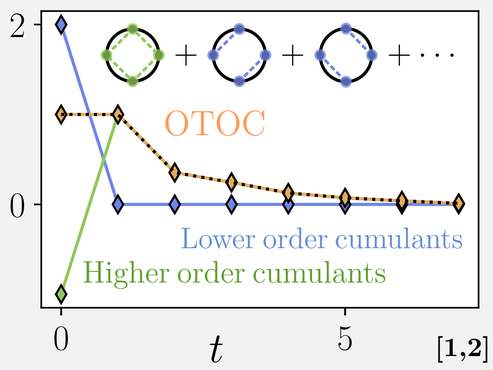
Eigenstate Thermalization Hypothesis (ETH)
Motivated by random matrix theory.
Local subsystems relax to equilibrium as the complement acts as large effective bath!

How do **multi-point correlation functions** relax to equilibrium?

Example: **Out of time ordered correlators** (OTOCs), $\langle A(t)AA(t)A\rangle$ in a chaotic circuit [1, 2]:



Identical and ergodic



Theoretical framework: **Full ETH** as the generalized ansatz

$$\overline{A_{i_1 i_2} A_{i_2 i_3} \ldots A_{i_q i_1}} = e^{-(q-1)S(E^+)} f^{(q)}_{E^+}(\omega_{i_1 i_2}, \ldots, \omega_{i_{q-1} i_q})$$

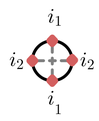
... and the combinatorics of **Free Probability Theory** (Non-commutative prob. theory)

Decomposition of correlation functions into **free cumulants:**

$$\kappa_q(\vec{t}) \equiv \frac{1}{D} \sum_{i_1 \neq i_2 \neq \ldots \neq i_q} A(t_1)_{i_1 i_2} A(t_2)_{i_2 i_3} \ldots A(0)_{i_q i_1}$$

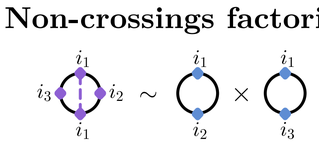
Free probability emerges in **thermodynamic limit** $D \to \infty$

Crossings are supressed



$$\equiv \frac{1}{D} \sum_{i_1 \neq i_2} \overline{|A_{i_1 i_2}|^2 |A_{i_1 i_2}|^2} \sim O\left(\frac{1}{D}\right).$$

Non-crossings factorize



Decay of OTOCs = **Free Independence** of $A(t)$ and A as $t \to \infty$

?

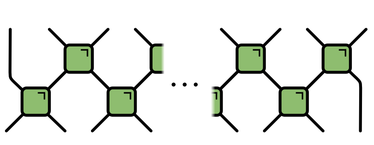
- Can we get analytical predictions for ETH diagrams?
- Does full ETH persist for **ergodicity breaking** in some form?

Non-ergodic: dual-unitary/XXZ circuit

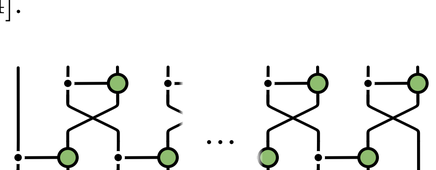
We consider a XXZ chain with local gates

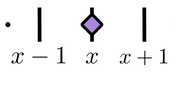
$$U = \exp\left[-i\tau \left(X \otimes X + Y \otimes Y + J_z\, Z \otimes Z\right)\right]$$

in a Brickwork configuration:

$U_F =$ 

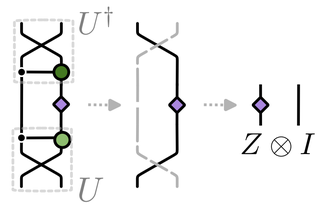
We first focus on the dual-unitary point, corresponding to an integrable circuit comprised of SWAP (denoted by ⨯) and controlled-phase (denoted by ┊—●) gates [4].

$U_F =$ 

Graphically, we introduce the Pauli operator: $Z_x =$ 

Solitons and charges

Solitons: operators which are shifted along the lattice under unitary evolution: $U_F^\dagger Z_x U_F = Z_{x+y}$

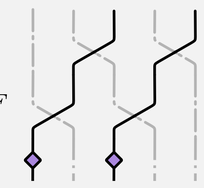


This feature leads to an exponential number of conserved charges:

$$Q_{(a)} = \sum_{t=0}^{L-1} \mathcal{S}^{-t} Z_{(a)} \mathcal{S}^t$$

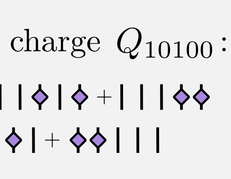
Example

$U_F^\dagger Z_{10100} U_F$ =



Z_{00101}

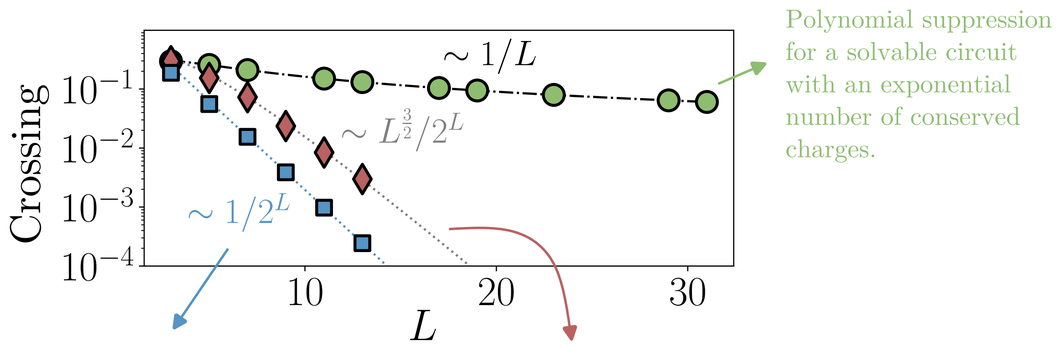
Two-body charge Q_{10100}:



The integrable model acts as a SWAP circuit upon the solitons. This construction leads to $2^L/L$ charges and symmetry sectors of dimension $L \times L$.

Crossing diagrams in full ETH

Full ETH holds in **integrable** models within symmetry sectors.
The combinatorics of Free Probability emerges slower with system size due to the large number of **conservation laws**.

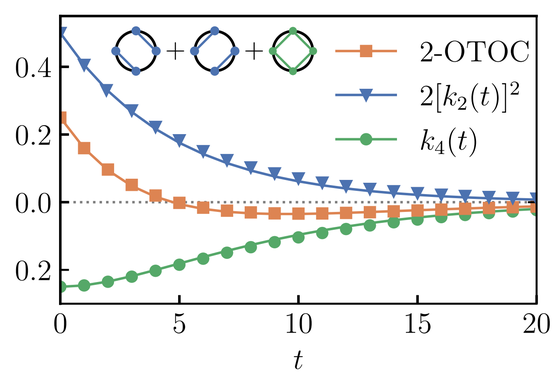


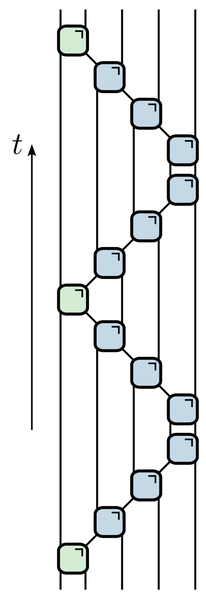
The charges in this model and its solvability allow us to obtain an analytical expression for crossing diagrams in the model:

$$C := \overline{|A_{ij}|^4} = \frac{2}{L} - \frac{4}{L^2} + \frac{2}{L^3}$$

Ergodic: Boundary scrambling

Boundary scrambling model: we introduce a solvable many-body circuit model where it is possible to obtain analytical predictions for higher order OTOCs and free cumulants [3, 5].





Results and Summary

- Full ETH within symmetry sectors in the presence of multiple conservation laws.
- Minimal circuit models for full ETH and free independence
- Novel analytical predictions for free cumulants and full ETH.

References

[1] Silvia Pappalardi, Felix Fritzsch, Tomaž Prosen, Physical Review Letters **134** (2025)
[2] Felix Fritzsch, Silvia Pappalardi, Tomaž Prosen, Physical Review B **111** (2025)
[3] Felix Fritzsch, Pieter W. Claeys, arXiv:2506.11197 (2025)
[4] Gabriel O. Alves, Felix Fritzsch, Pieter W. Claeys, arXiv:2504.08517 (2025)
[5] Felix Fritzsch, Gabriel O.Alves, Michael A. Rampp, Pieter W. Claeys, arXiv:2509.08060