Critical properties of the Rabi model

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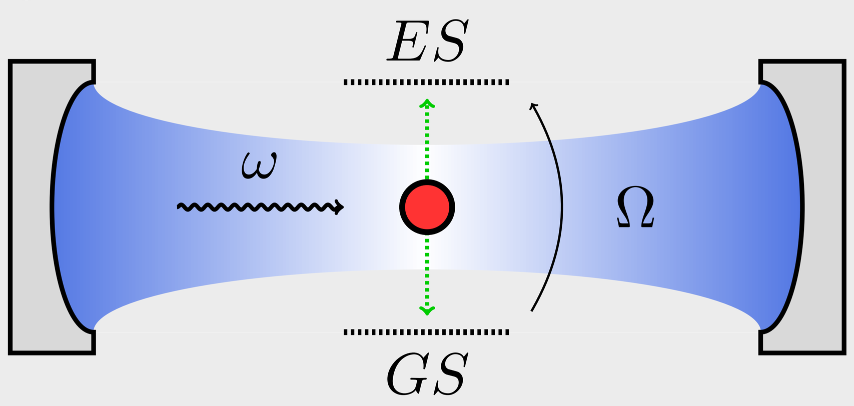
Introduction

The Quantum Rabi model describes the interaction of a quantized field with a two-level atom and is characterized by:

$$H_{Rabi} = \omega_0 a^\dagger a + \frac{\Omega}{2}\sigma_z - \lambda(a + a^\dagger)\sigma_x \qquad (1)$$

In the present work we investigate the critical properties of the critical Rabi model, in and out of equilibrium. In particular, the nature of the phase transition for the model and the dynamics of relaxation under a linear quench around the critical point, reproducing the results from.[1]



Schrieffer-Wolf Transformation

Since the interaction term $\lambda(a + a^\dagger)\sigma_x$ introduces off-diagonal block matrices in the Hamiltonian, our first task is to find a method to diagonalize it. As it was done in,[1] the procedure that we'll employ is the Schrieffer-Wolff transformation. The method yields the following generator:

$$S = \frac{1}{\Omega}(a + a^\dagger)(\sigma_+ - \sigma_-) \qquad (2)$$

In the thermodynamic limit a transformation given by this generator produces a gaussian Hamiltonian:

$$\tilde{H}_{np} = e^{-S} H_{Rabi} e^{S} = \omega_0 a^\dagger a - \frac{\Omega}{2} - \frac{\omega_0 g^2}{4}(a + a^\dagger)^2 \qquad (3)$$

Where the coupling constant g is given by $g = 2\lambda/\sqrt{\omega_0\Omega}$.

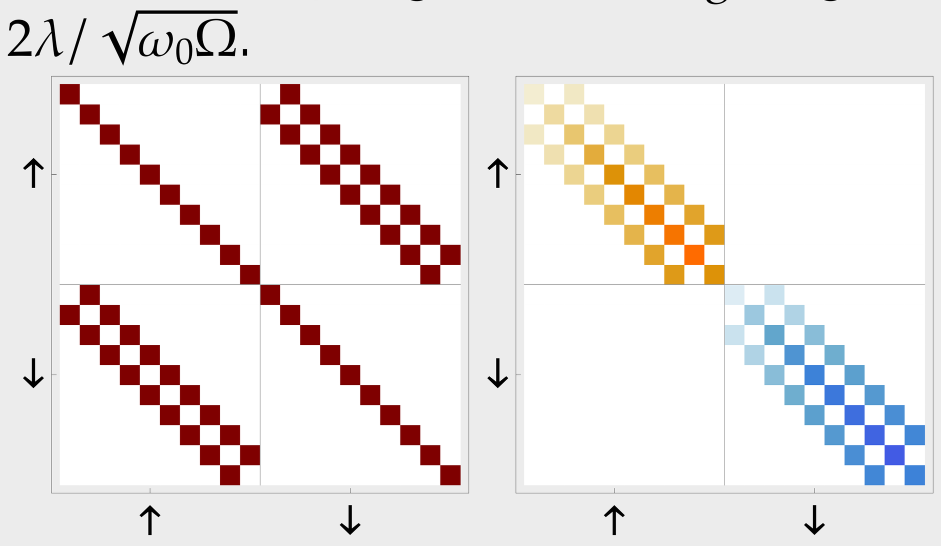


Figure 1: A representation of the Hamiltonian in matrix form before and after the transformation up to first order. The blue block matrix represents the low-energy subspace that we obtain after an projection in that subspace.

Diagonalized Hamiltonian

With a quadratic Hamiltonian in our hands we can perform a Bogoliubov transformation on the bosonic operators using the squeezing operator, with an appropriate choice of parameters this will give us a diagonal Hamiltonian:

$$H_{np} = S^\dagger(r_{np})\tilde{H}_{np}S(r_{np}) = \omega_0\sqrt{1-g^2}\,a^\dagger a - \frac{\Omega}{2} + \frac{\epsilon_{np} - \omega_0}{2} \qquad (4)$$

With $r_{np} = -1/4\ln(1-g^2)$. This equation however fails for $g_c = 1$, since the energy gap closes at this critical point. If we dislocate the Hamiltonian first, using the displacement operator $D(\alpha) = \alpha a^\dagger - \alpha^* a$ we can bypass this problem. With an adequate choice for the displacement parameter, given by $\alpha = \sqrt{\Omega/4\omega_0(g^2 - g^{-2})}$. We can apply the squeezing operators once again, this time with the choice $r_{sp} = -1/4\ln(1-g^{-2})$. The result is:

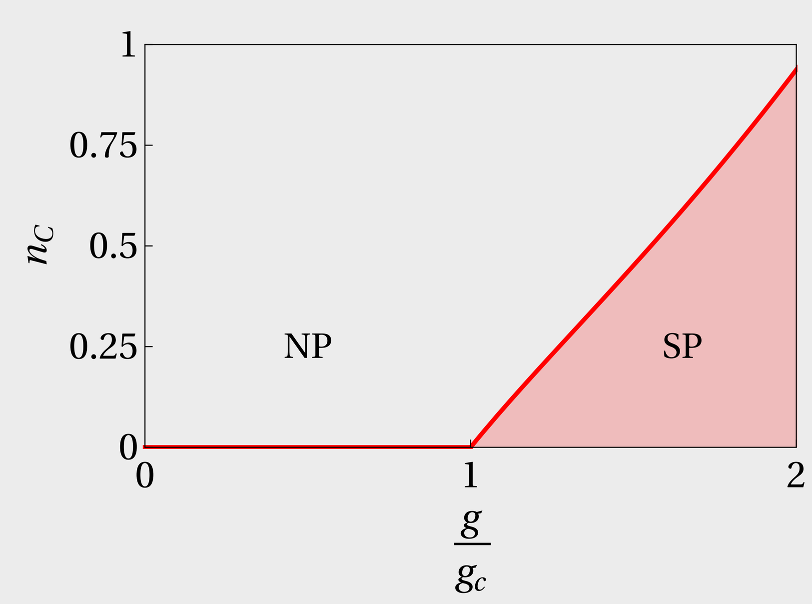
$$H_{sp} = \omega_0\sqrt{1-g^{-2}}\,a^\dagger a + \frac{\epsilon_{np} - \omega_0}{2} - \frac{\Omega}{4}(g^2 + g^{-2}) \qquad (5)$$

Phase Transition

The order parameter for this phase transition is the normalized number of photons, given by:

$$n_c = \begin{cases} 0, & g \leq 1 \\ \frac{g^2 - g^{-2}}{4}, & g > 1 \end{cases}$$

The phase for $g > 1$ is called the *superradiant phase*, due to the macroscopic occupation of n_c.



This is a second order phase transition, as shown in the picture below.

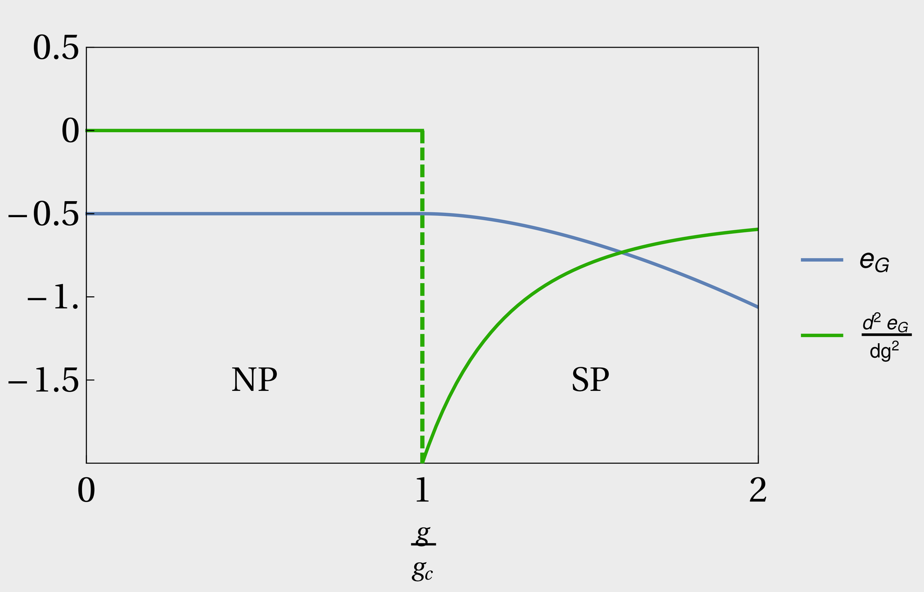


Figure 2: Ground state energy of the system and its second derivative as a function of g/g_c. Note that there's a discontinuity at the critical point for the second derivative of the GS energy. This characterizes the 2nd order phase transition.

Relaxation Dynamics

Lastly we study the dynamics of the system after a sudden linear quench in the parameter g, given by $g(t) = g_f t/\tau_q$. The system behaves adiabatically when far away from the critical point, and the residual energy, defined as $E_r(t) = \langle 0|H(t)|0\rangle - E_G(t)$, scales with τ_q^{-2}. On the other hand, near the critical point the system behaves impulsively, and the Kibble Zurek Mechanism predicts a scaling of $\tau_q^{-1/3}$. These relations could be confirmed with numerical simulations, as shown below.

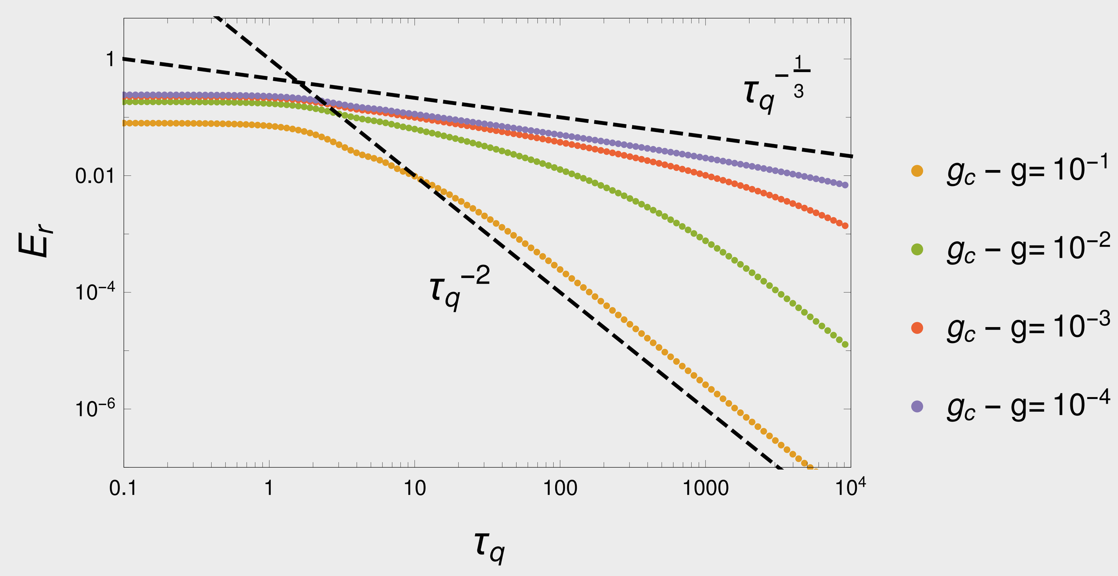


Figure 3: Residual energy as a function of the final quench time. The dashed lines show the expected behavior for both regimes. The colored lines represent the numerical solutions for the system Dynamics.

Dissipative Phase Transition

The dissipation in the model, which corresponds to a possible loss of photons in the cavity, can be described through a master equation of the form

$$\dot{\rho} = \mathcal{L}[\rho] = -i[H_{Rabi}, \rho] + 2\kappa\mathcal{D}[a] \qquad (6)$$

with a dissipator of the form $\mathcal{D}[a] = a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a$. This dissipator generates the Liouvillian eigenvalue at the normal phase:

$$\ell_{np} = -\kappa \pm i\omega_0\sqrt{1-g^2} \qquad (7)$$

In the closed QRB, the phase transition is characterized by the closing of the energy gap. However, in the dissipative case we observe the closing of the Liouvillian gap instead, which occurs at the critical point:

$$g_c = \sqrt{1 + \kappa^2/\omega_0^2} \qquad (8)$$

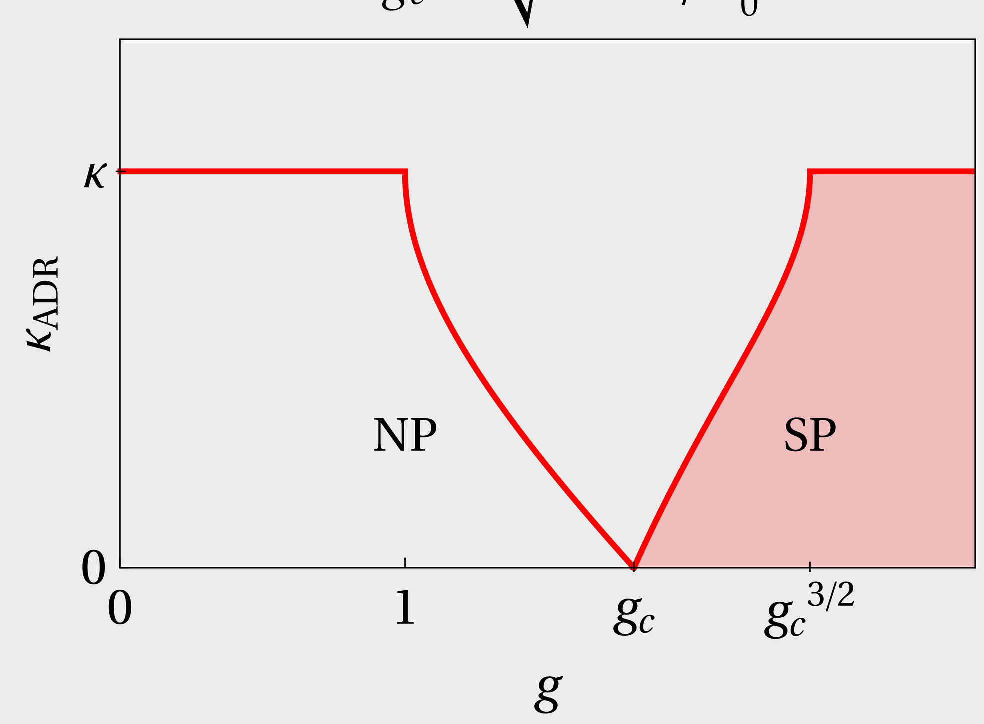


Figure 4: Plot of the *Asymptotic Decay Rate* (ADR), defined as the real part of the Liouvillian eigenvalues. To find the eigenvalues ℓ_{sp} for the superradiant phase we simply substitute $g \to g^3/g_c^3$ into ℓ_{np}.

Conclusion and Acknowledgements

We were able to unveil many of the properties of the model, in and out of equilibrium. Investigations of the system in the context of weak measurements could be made in the future. The authors acknowledge the financial support from FAPESP, project number 2018/09472-6.

References

[1] Myung-Joong Hwang, Ricardo Puebla, and Martin B. Plenio. Quantum phase transition and universal dynamics in the rabi model. *Physical Review Letters*, 115(18), oct 2015.
[2] Christopher Gerry and Peter Knight. *Introductory Quantum Optics*. Cambridge University Press, 1 edition, 2004.