mpipks

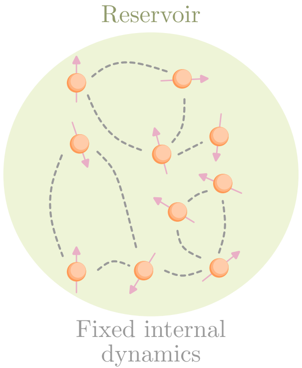
A (very) short introduction to Quantum Reservoir Computing

Gabriel O. Alves

June 24, 2026

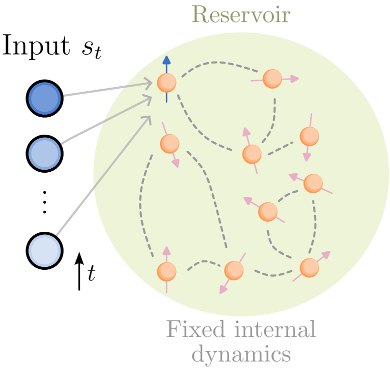
Starting point: Can we harness the *inherent* complexity and internal dynamics of quantum systems for learning?

Quantum Reservoir Computing



Quantum Reservoir Computing

Input $s_t \rightarrow$ Reservoir



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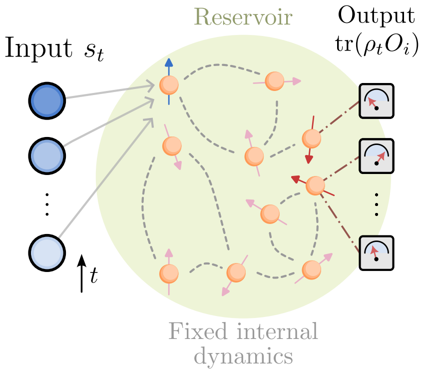
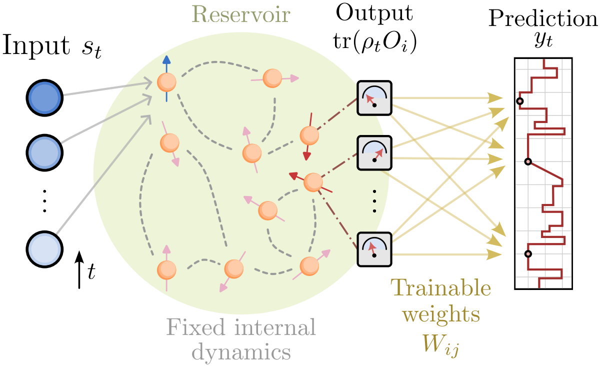
Input $s_t \rightarrow$ Reservoir $\rightarrow$ Measure observables O_i



Quantum Reservoir Computing

Input $s_t \to$ Reservoir $\to$ Measure observables $O_i \to$ Prediction y_t



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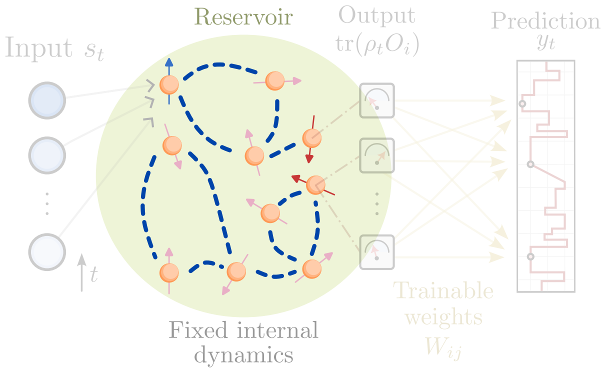

Acts as a fixed/untrainable hidden layers

Quantum Reservoir Computing

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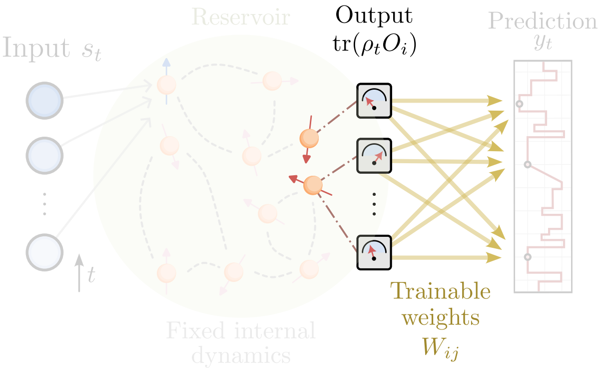
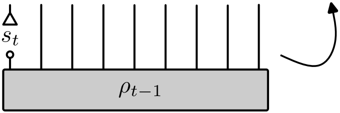


Single trainable/visible layer: simple regression

Quantum Reservoir Computing - 1. Encoding

Input encoding via reset-and-inject protocol by Fujii and Nakajima [1]

$$\mathcal{E}(\rho_{t-1}) := |s_t\rangle \langle s_t| \otimes \mathrm{PTr}(\rho_{t-1})$$



[1] [Physical Review Applied 8, 024030]

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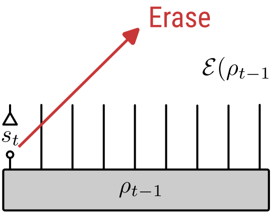
1. Reset the first qubit



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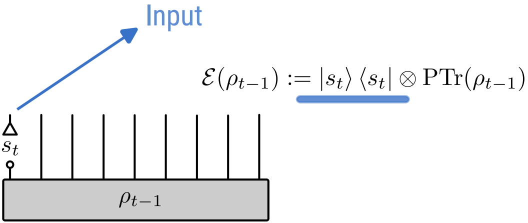
Quantum Reservoir Computing - 1. Encoding

Input encoding via reset-and-inject protocol by Fujii and Nakajima [1]

1. Reset the first qubit
2. Encode the external input $|s_t\rangle = \sqrt{1 - s_t}\,|0\rangle + \sqrt{s_t}\,|1\rangle$



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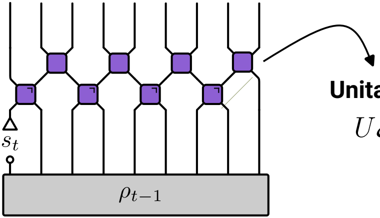
Concrete reservoir example: **brickwork quantum circuit**



Unitary evolution:

$U\mathcal{E}(\rho_{t-1})U^\dagger$

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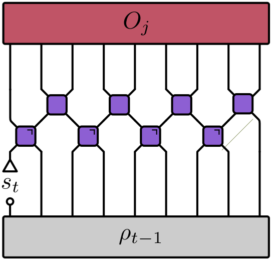
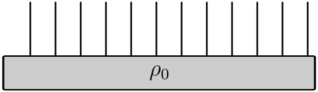


Features: expected values

$$\mathrm{Tr}(\rho O_j) =: x_{t,j}$$

[1][Physical Review Applied 8, 024030]

Constructing the feature matrix



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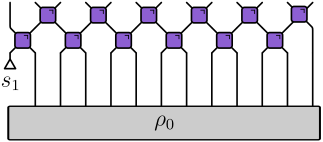

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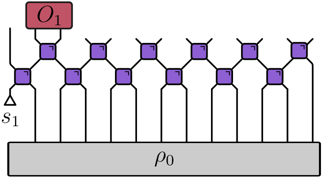
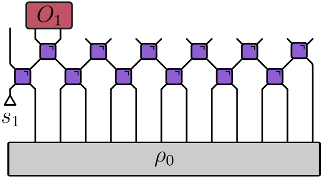


Constructing the feature matrix

$$\mathbf{x} = \left(\quad \right)$$

Compute expected value at time t

$$x_{t,j} = \langle O_j \rangle_t = \mathrm{Tr}(\rho_t O_j)$$



Constructing the feature matrix

$$\mathbf{x} = \begin{pmatrix} x_{11} & & \\ & & \\ & & \end{pmatrix}$$

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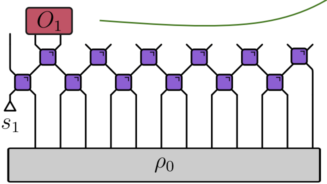
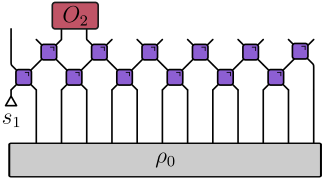


Constructing the feature matrix

$$\mathbf{x} = \begin{pmatrix} x_{11} & x_{12} & & \\ & & & \\ & & & \end{pmatrix}$$

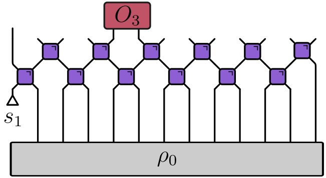
We compute **another** observable



Constructing the feature matrix

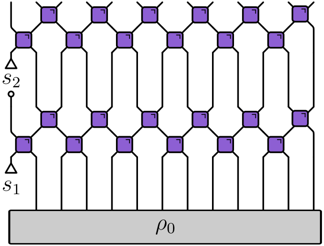
$$\mathbf{x} = \begin{pmatrix} x_{11} & x_{12} & x_{13} & \cdots \\ & & & \\ & & & \end{pmatrix}$$

and so on...



Constructing the feature matrix

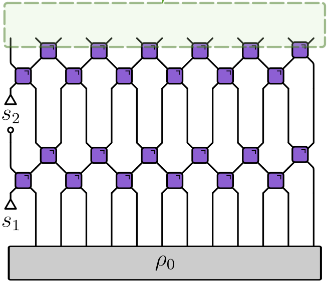
Insert new input and circuit layer



$$\mathbf{x} = \begin{pmatrix} x_{11} & x_{12} & x_{13} & \cdots \\ & & & \\ & & & \end{pmatrix}$$

Constructing the feature matrix

Compute expectations at t = 2



$$\mathbf{x} = \begin{pmatrix} x_{11} & x_{12} & x_{13} & \cdots \\ x_{21} & x_{22} & x_{23} & \cdots \\ & & & \end{pmatrix}$$

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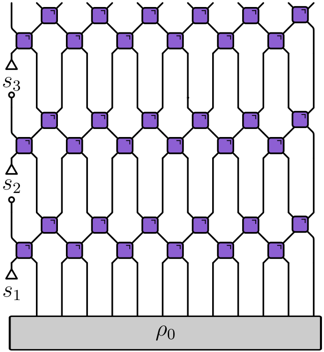

$$\mathbf{x} = \begin{pmatrix} x_{11} & x_{12} & x_{13} & \cdots \\ x_{21} & x_{22} & x_{23} & \cdots \\ x_{31} & x_{32} & x_{33} & \cdots \\ & & & \end{pmatrix}$$

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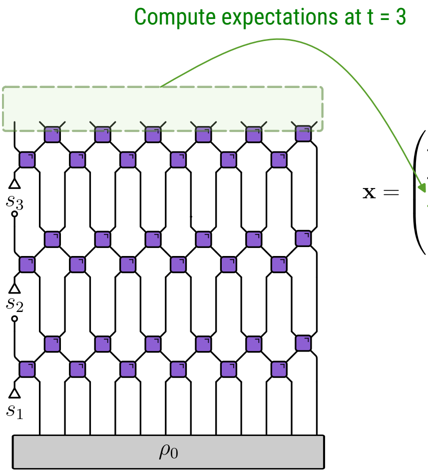

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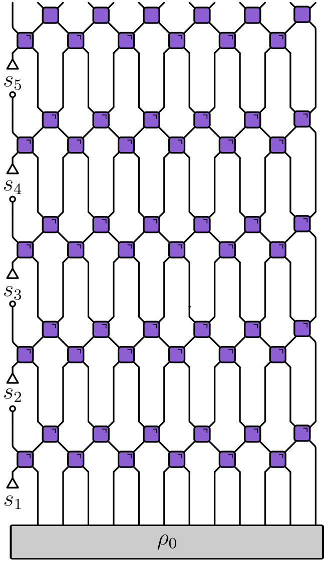

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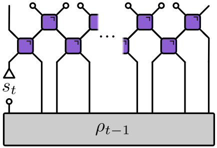
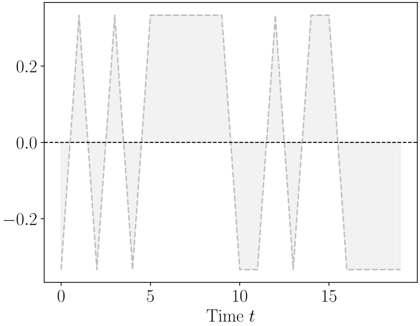
1. Erase-input encoding - concrete example

Consider a numerical example for the dynamics and a few observables:





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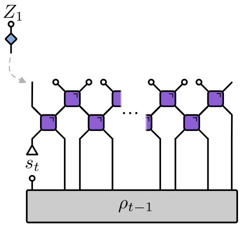

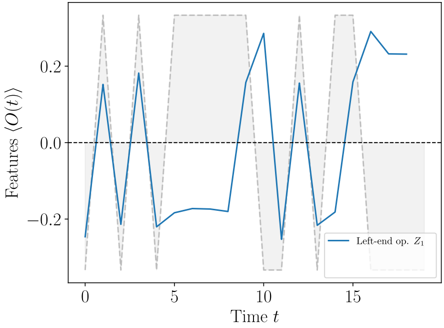

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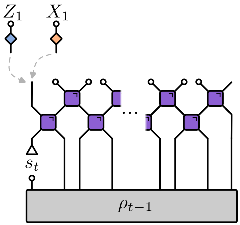

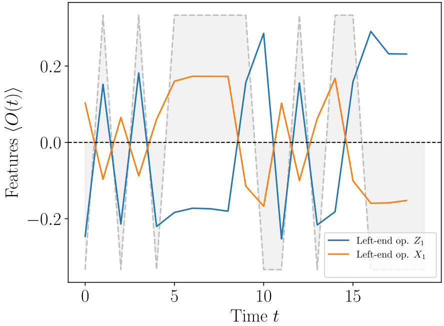

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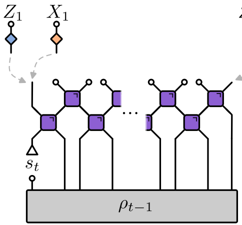

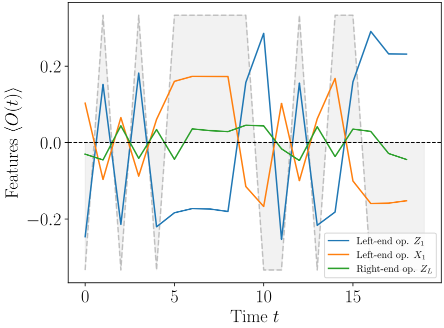

2. Benchmark tasks

Reservoirs need to capture **memory and nonlinearity**

☞ Short-term memory with time-lag τ:

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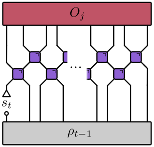
By tuning τ we will probe the memory of the reservoir.

3. Training

Given the features $x_{t,j} = \mathrm{tr}\{\rho_t O_j\} =$ 

:

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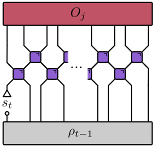
Given the features $x_{t,j} = \mathrm{tr}\{\rho_t O_j\} =$  :

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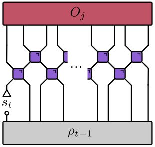
Given the features $x_{t,j} = \mathrm{tr}\{\rho_t O_j\} =$  :

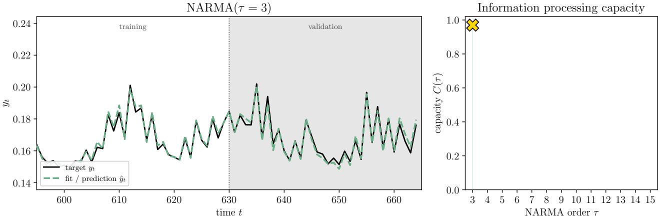
- Ridge regression: $\mathbf{W} = (\mathbf{x}^T\mathbf{x} + \alpha\mathbf{I})^{-1}\mathbf{x}^T\mathbf{y}$
- Prediction: $\mathbf{y} = \mathbf{W}\mathbf{x}$
- We evaluate performance through the information processing capacity (IPC): $C = \frac{\mathrm{cov}^2(\mathbf{y}_{\mathrm{val}}, \mathbf{y}_{\mathrm{tar}})}{\mathrm{var}(\mathbf{y}_{\mathrm{val}})\mathrm{var}(\mathbf{y}_{\mathrm{tar}})}$

Learning in practice: trying to learn a NARMA time-series with a small brickwork circuit as a reservoir.

Learning the NARMA(τ) task

Concrete numerics for $L = 7$ and 39 nodes

$$\text{NARMA}(\tau) \text{ task: } y_k = 0.3\, y_{k-1} + 0.05\, y_{k-1} \sum_{i=1}^{\tau} y_{k-i} + 1.5\, s_{k-\tau}\, s_{k-1} + 0.1$$



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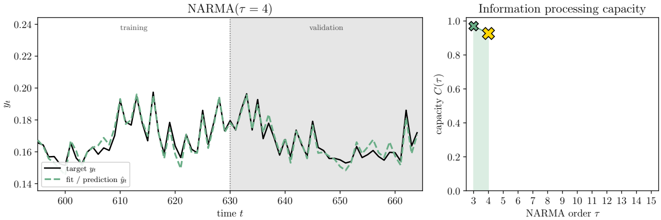

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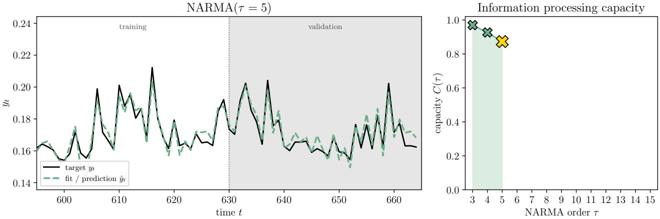

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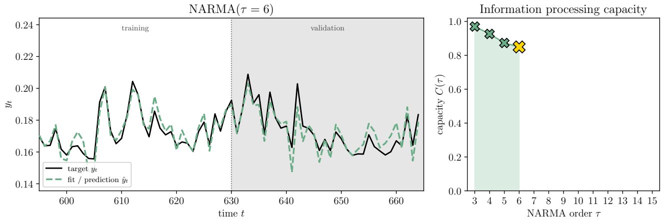

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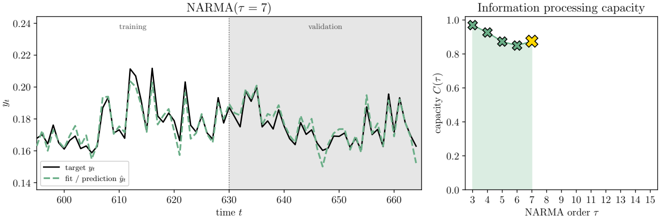

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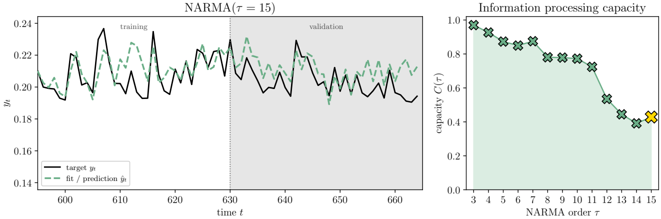
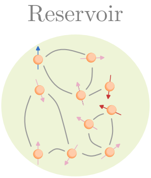


Quantum Reservoir Computing - Summary



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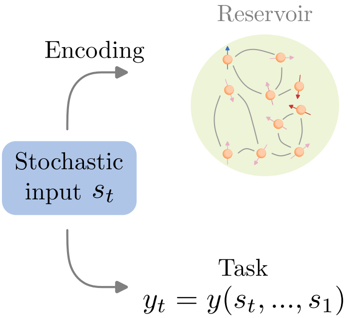

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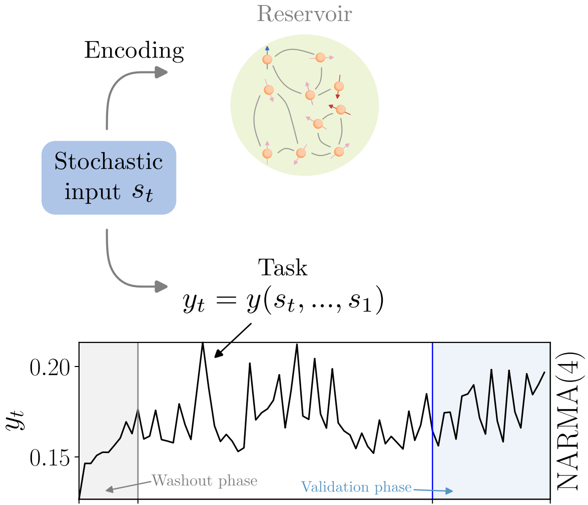

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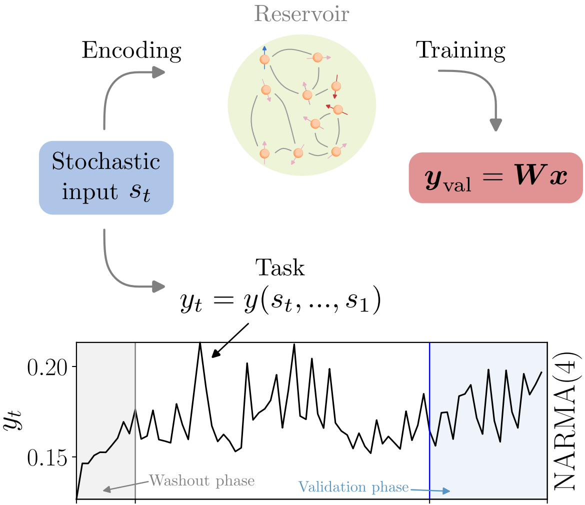

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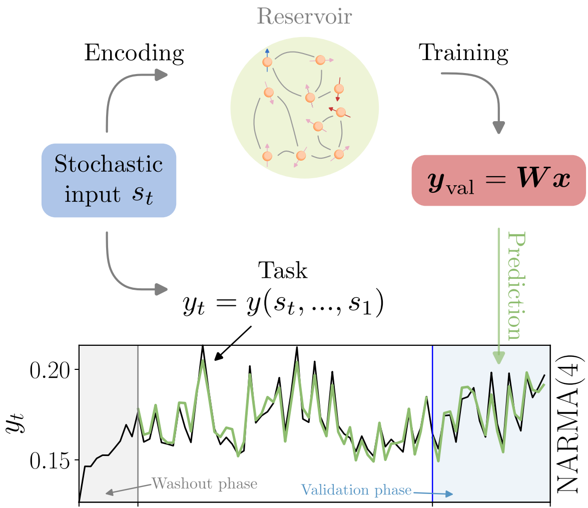
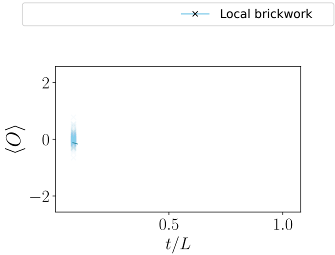
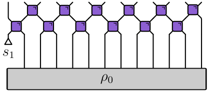


What are the limitations of QRC?

The problem of exponential concentration





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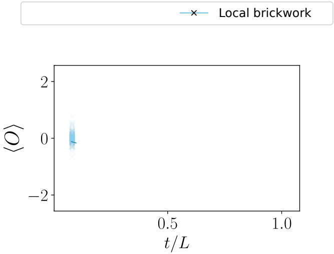
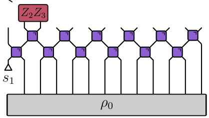


We pick some arbitrary local operator and plot its expectation values



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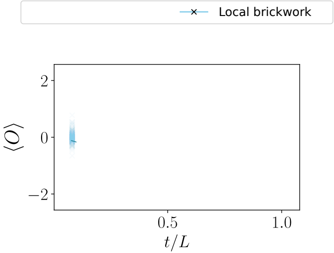
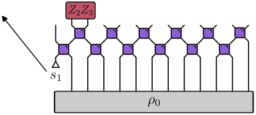
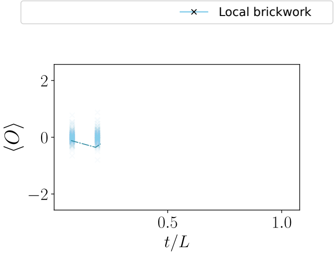
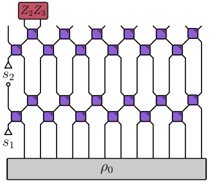


Repeat the process
for many stochastic
inputs $s_1 \in [0, 1]$



The problem of exponential concentration





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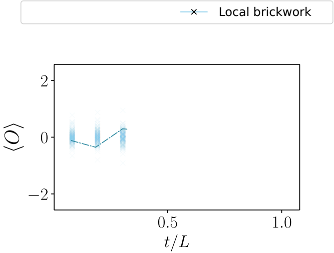

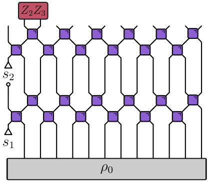

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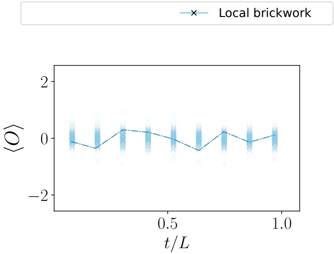
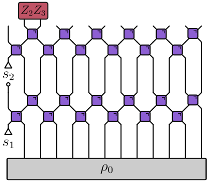
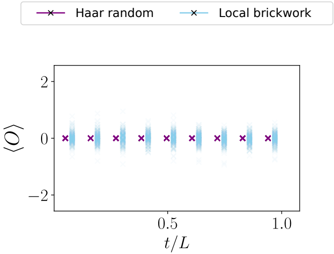
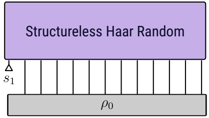


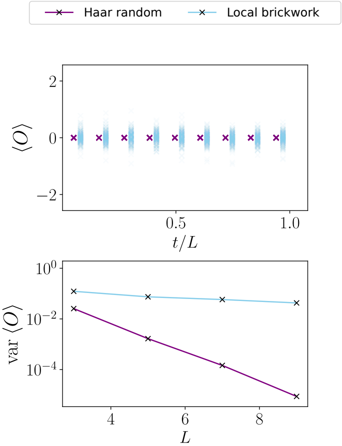


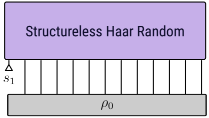
The problem of exponential concentration





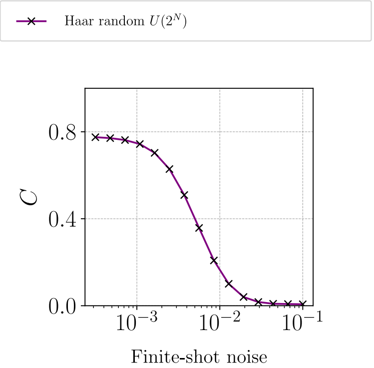
The problem of exponential concentration





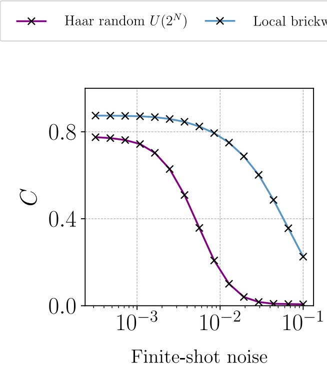
The problem of exponential concentration

Finite-shot noise degrades learning!



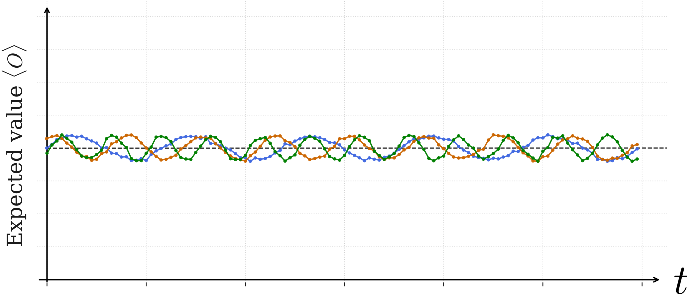
The problem of exponential concentration

Finite-shot noise degrades learning! Previous slide: Local structure can help [2]



[2] W. Xiong, Z. Holmes, A. Angrisani, Y. Suzuki, T. Chotibut, and S. Thanasilp, “Role of scrambling and noise in temporal information processing with quantum systems,” arXiv:2505.10080 (2025).

Interpretation



Interpretation

Conclusion

- ☞ We only discussed one reservoir architecture; many other physical platforms are available (e.g. photonics, Bose-Hubbard lattices, ...)

  G. Llodrà, P. Mujal, R. Zambrini, and G. L. Giorgi, “Quantum reservoir computing in atomic lattices,” arXiv:2411.13401 (2024).

- ☞ Rigorous approaches to learning theory: what conditions are necessary for QRC to work?

  A. Sannia, R. Martínez-Peña, M. C. Soriano, G. L. Giorgi, and R. Zambrini, “Dissipation as a resource for Quantum Reservoir Computing,” Quantum 8, 1291 (2024).

- ☞ Interplay between physical properties and learning performance (e.g. chaos, thermalization, and many-body localization)

  R. Martínez-Peña, G. L. Giorgi, J. Nokkala, M. C. Soriano, and R. Zambrini, “Dynamical phase transitions in quantum reservoir computing,” Physical Review Letters 127, 100502 (2021).