Time-optimal holonomic quantum computation

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Outline

Introduction

How can we design robust quantum gates?

A concrete proposal: the Λ-system

- ☞ Based on non-adiabatic non-abelian geometrical phases
- ☞ Robust against certain types of noise & dissipation
- ☞ Can implement a universal set of gates
- ☞ Experimentally friendly

Geometrical character and short operation times → robust gates.

New Journal of Physics

The open-access journal for physics

Non-adiabatic holonomic quantum computation

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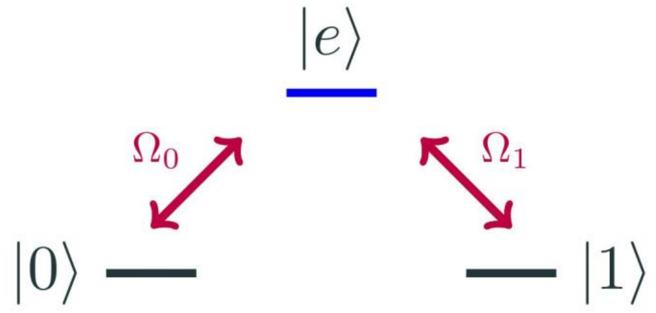

This work

The problem: breakdown in the rotating wave approximation (RWA) for short operation times $\rightarrow$ counter-rotating effects become significant.

PHYSICAL REVIEW A **88**, 054301 (2013)

Validity of the rotating-wave approximation in nonadiabatic holonomic quantum computation

Jakob Spiegelberg[1] and Erik Sjöqvist[1,2]
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[2]*Centre for Quantum Technologies, National University of Singapore, 3 Science Drive 2, 117543 Singapore, Singapore*
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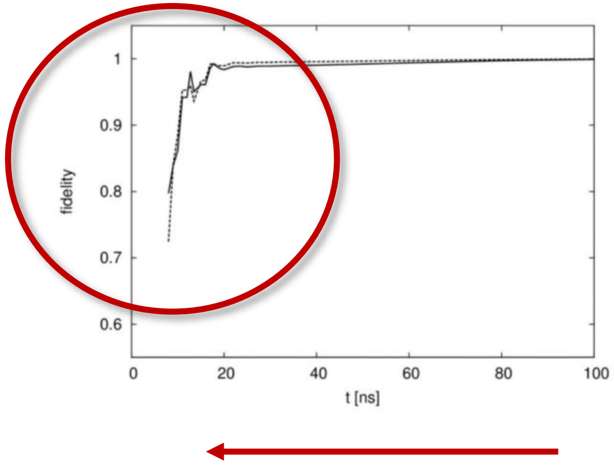

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Our objective: to study the trade-off between dissipative effects and the validity of the RWA

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Time-optimal holonomic quantum computation

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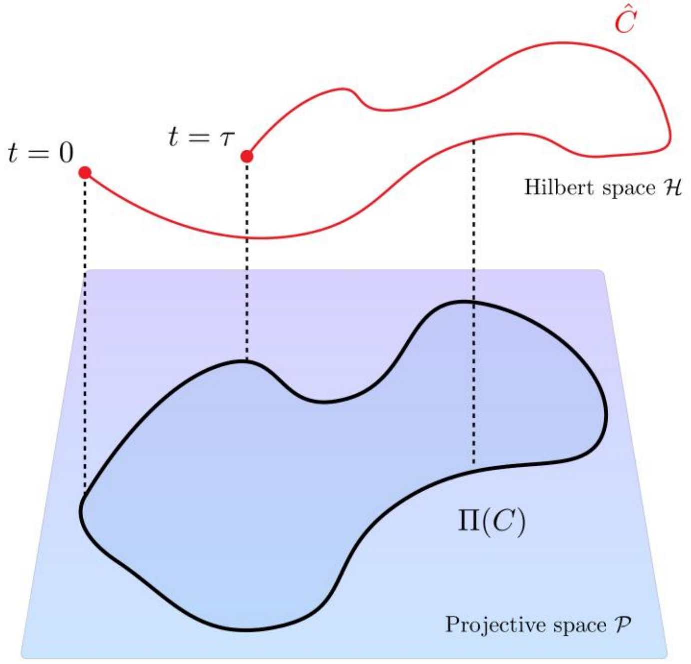
Geometrical Phases

Non-adiabatic non-Abelian geometrical phase

Berry, 1980s: geometrical phases in *adiabatic* evolutions.

- ☞ Can we extend this to the non-adiabatic case? [1]
- ☞ Can we get a non-abelian structure?[2]



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[1] Y. Aharonov and J. Anandan, Phys. Rev. Lett. **58**, 1593 (1987)

[2] J. Anandan, Phys. Lett. A **133**, 171 (1988).

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Non-abelian non-adiabatic evolution

The unitary is given by:

$$\boldsymbol{U}(t) = \mathcal{T}\exp\left\{\left(\int_0^t i(\boldsymbol{A} - \boldsymbol{K})dt\right)\right\}$$

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Vanishing $\boldsymbol{K}$: $\boldsymbol{U}(C) = \mathcal{P} \exp\left\{ \left(i \oint_C \mathcal{A} \right) \right\}$, with $\mathcal{A} := i\langle \tilde{\psi}_a | d | \tilde{\psi}_b \rangle$

Tha Λ-system satisfies these conditions!

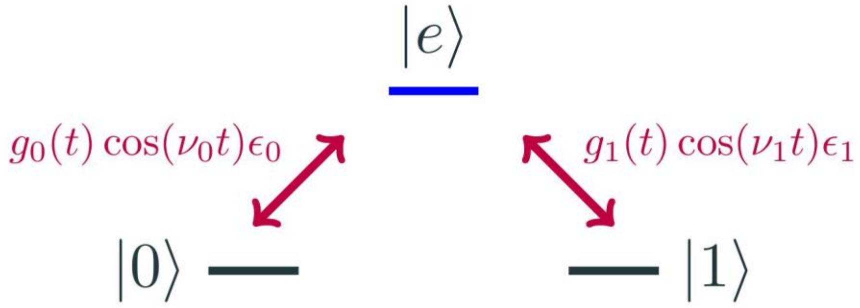
The Λ-system

The Λ-system

Consider a three-level system with $H(t) = H_0 + \boldsymbol{\mu} \cdot \boldsymbol{E}(t)$, where:[1]

$$\boldsymbol{E}(t) = g_0(t)\cos(\nu_0 t)\boldsymbol{\epsilon}_0 + g_1(t)\cos(\nu_1 t)\boldsymbol{\epsilon}_1,$$

and $H_0 = -f_{e0}\left|0\right\rangle\left\langle 0\right| - f_{e1}\left|1\right\rangle\left\langle 1\right|$.



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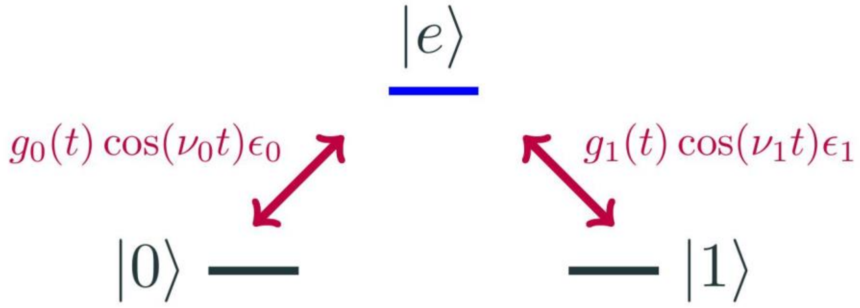
Interaction picture:

$$H_I(t) = e^{-iH_0 t} H(t) e^{iH_0 t}$$



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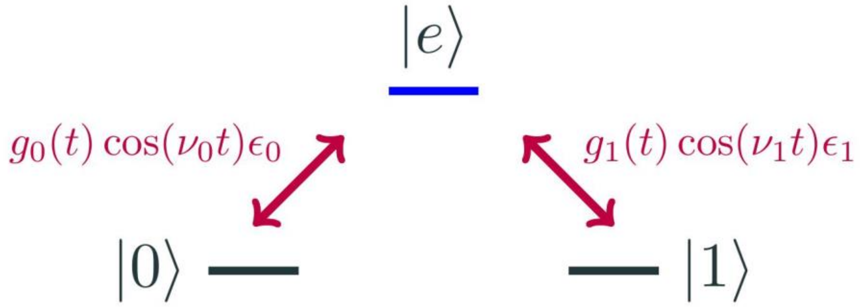

Interaction picture:

$$H_I(t) = e^{-iH_0 t} H(t) e^{iH_0 t}$$

We get:

$$\begin{aligned} H_I(t) &= \Omega_0(t)(e^{-i(f_{e0}+\nu_0)t} + e^{-i(f_{e0}-\nu_0)t})\left|e\right\rangle\left\langle 0\right| \\ &+ \Omega_1(t)(e^{-i(f_{e1}+\nu_1)t} + e^{-i(f_{e1}-\nu_1)t})\left|e\right\rangle\left\langle 1\right| + \text{h.c} \end{aligned}$$

Take resonant frequencies $\nu_i = f_{ej}$

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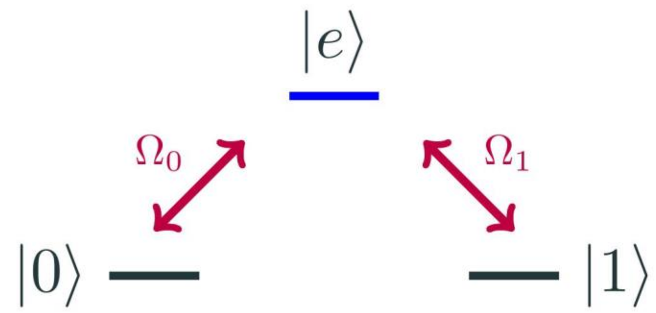
The Λ-system

The Hamiltonian of the Λ-system is:

$$H_I(t) = \Omega_0(t)(1 + e^{-2if_{e0}t}) |e\rangle \langle 0| + \Omega_1(t)(1 + e^{-2if_{e1}t}) |e\rangle \langle 1| + \text{h.c.}$$

Where $\Omega_j = \langle e| \boldsymbol{\mu} \cdot \boldsymbol{\epsilon} |j\rangle g_j(t)/2$ are transition frequencies



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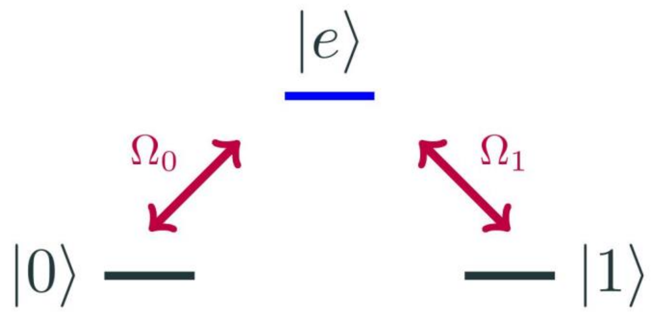


☞ How to handle counter-rotating terms of the type:

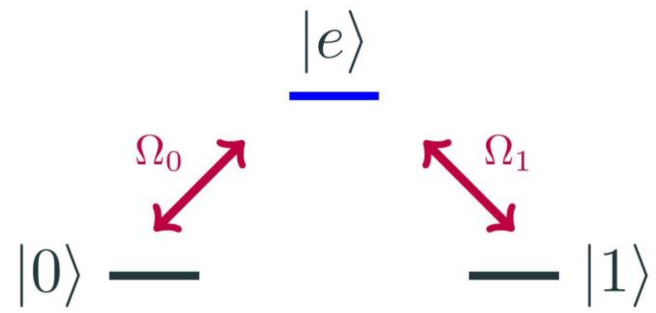
$$(1 + e^{-2if_{ej}t})?$$

Standard strategy: Rotating wave approximation (RWA)

Bright and dark states

The RWA Hamiltonian is:

$$H_I^{RWA}(t) = \Omega_0(t) |e\rangle \langle 0| + \Omega_1(t) |e\rangle \langle 1| + \text{h.c}$$



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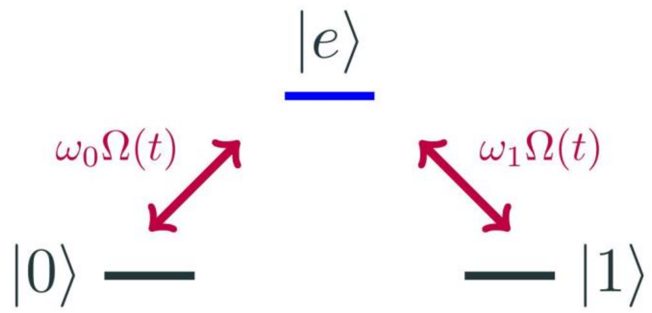
Two eigenstates:

$$\begin{aligned}\left|d\right\rangle =& \omega_0\left|1\right\rangle - \omega_1\left|0\right\rangle \\ \left|b\right\rangle =& \omega_0^*\left|0\right\rangle + \omega_1^*\left|1\right\rangle\end{aligned}$$



Also define $\Omega_j(t) = \omega_j\Omega(t)$, with $|\omega_0|^2 + |\omega_1|^2 = 1$, to obtain:

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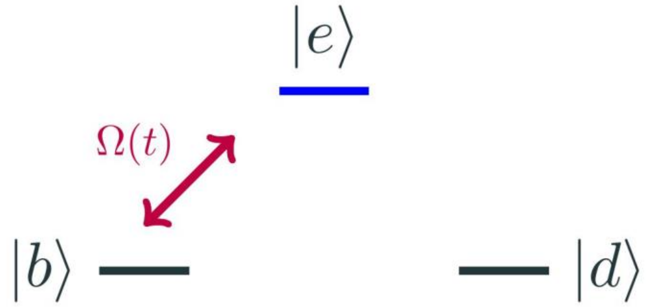

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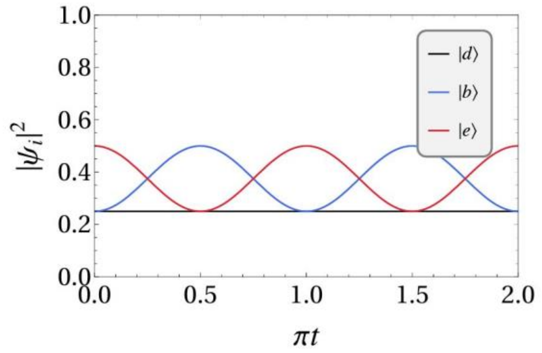
$$H_I^{RWA} = \Omega(t)(|e\rangle\langle b| + |b\rangle\langle e|)$$



$|d\rangle$ is decoupled!

Holonomic Quantum gate

Unitary in $\{|b\rangle, |d\rangle, |e\rangle\}$ (for $\Phi := \int_0^t \Omega(t')dt' = \pi$):

$$\mathcal{U}_{bd}(t,0) = |d\rangle\langle d| - |b\rangle\langle b| - |e\rangle\langle e|$$

Projecting onto the computational subspace $\{|0\rangle, |1\rangle\}$:

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$$U(C) = \begin{pmatrix} \cos\theta & e^{-i\phi}\sin\theta \\ e^{i\phi}\sin\theta & -\cos\theta \end{pmatrix} = \boldsymbol{n}\cdot\boldsymbol{\sigma}$$

- ☞ With $\boldsymbol{n} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$
- ☞ Parametrization: $\omega_0 = \sin(\theta/2)e^{i\phi}$ and $\omega_1 = -\cos(\theta/2)$

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Arbitrary single gate operations require two loops C_n and C_m:

$$U(C) = U(C_{\boldsymbol{m}})U(C_{\boldsymbol{n}}) = \boldsymbol{n}\cdot\boldsymbol{m} - i\boldsymbol{\sigma}\cdot(\boldsymbol{n}\times\boldsymbol{m}).$$

Rotation: plane spanned by $\mathbf{n}$ and $\mathbf{m}$, by an angle $2\cos^{-1}(\mathbf{n}\cdot\mathbf{m})$

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Implement a gate by choosing θ and ϕ

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Arbitrary single gate operations require two loops C_n and C_m:

E. g. $\theta = \pi/4$ and $\phi = 0$ for Hadamard

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Rotation: plane spanned by $\mathbf{n}$ and $\mathbf{m}$, by an angle $2\cos^{-1}(\mathbf{n}\cdot\mathbf{m})$

Results: dropping the RWA

The non-ideal scenario

$$H_I(t) = \omega_0 \Omega(t)(1 + e^{-2if_{e0}t}) \left|e\right\rangle \left\langle 0\right| + \omega_1 \Omega(t)(1 + e^{-2if_{e1}t}) \left|e\right\rangle \left\langle 1\right| + \text{h.c.}$$

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Counter-rotating terms

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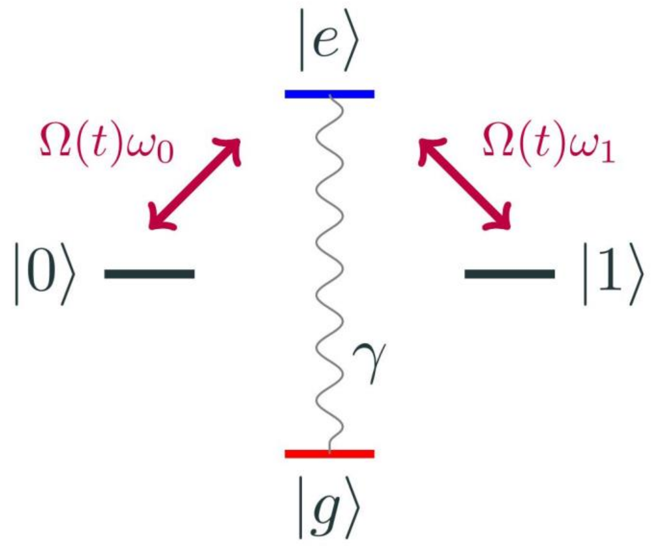
☞ How to also include dissipation?

$$\frac{d\rho}{dt} = i[\rho, H_i(t)] + \gamma D(\rho)$$

$D[\rho]$ is a dissipator and $L = |g\rangle\langle e|$:

$$D(\rho) = L\rho L^\dagger - \frac{1}{2}\{L^\dagger L, \rho\}$$



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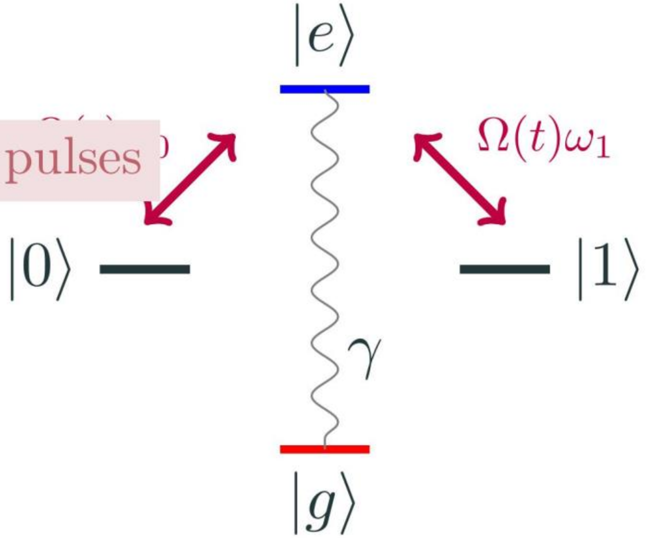

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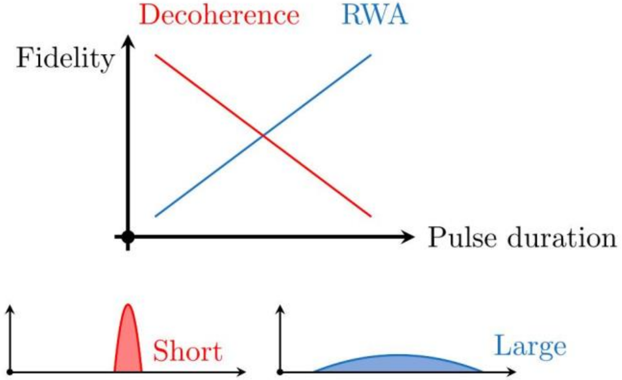
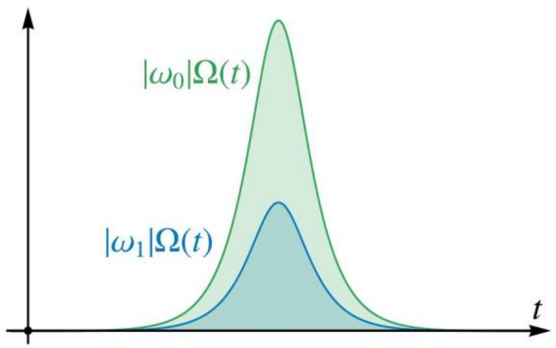


Simulations: the recipe

☞ Sensible choice of pulse envelope $\Omega(t) = \beta\,\mathrm{sech}(\beta t)$



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- ☞ Sensible choice of pulse envelope $\Omega(t) = \beta\,\mathrm{sech}(\beta t)$
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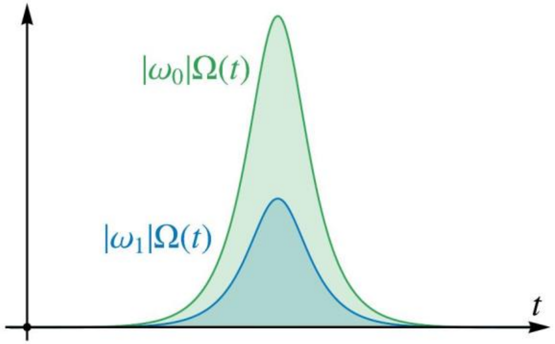

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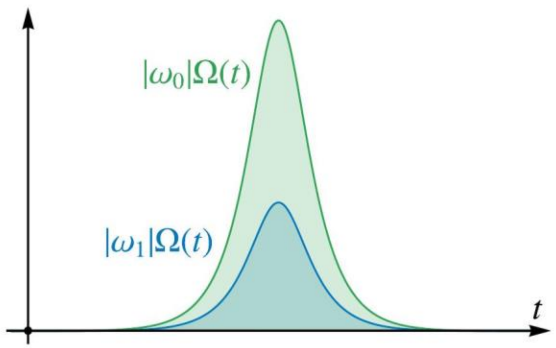

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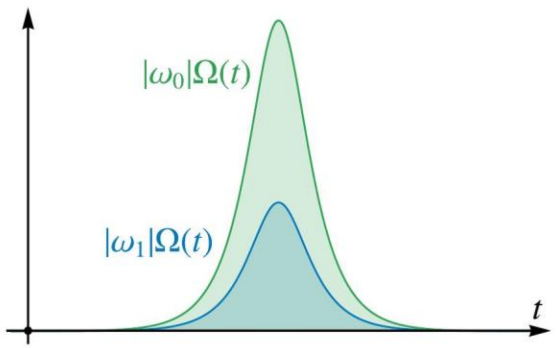

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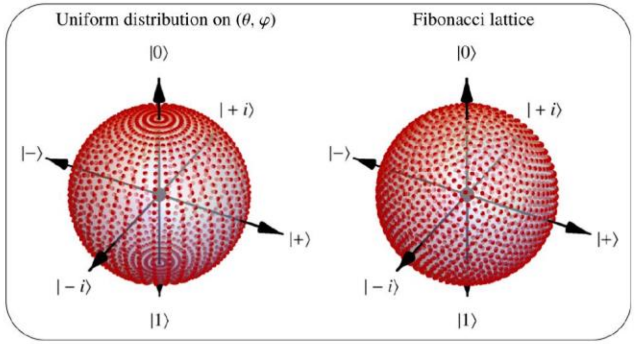
- ☞ Repeat the process for points sampled uniformly in the Bloch sphere [1]



[1] D. P. Hardin, T. J. Michaels, and E. B. Saff, Dolomites Res. Notes Approx. **9**, 16 (2016).

Results - single qubit gates

Can we find the optimal β for given γ and f_i?

- ☞ Shorter pulses (larger β) $\rightarrow$ counter-rotating frequencies f_{ej} dominate
- ☞ Longer pulses (smaller β) $\rightarrow$ dissipative effects dominate

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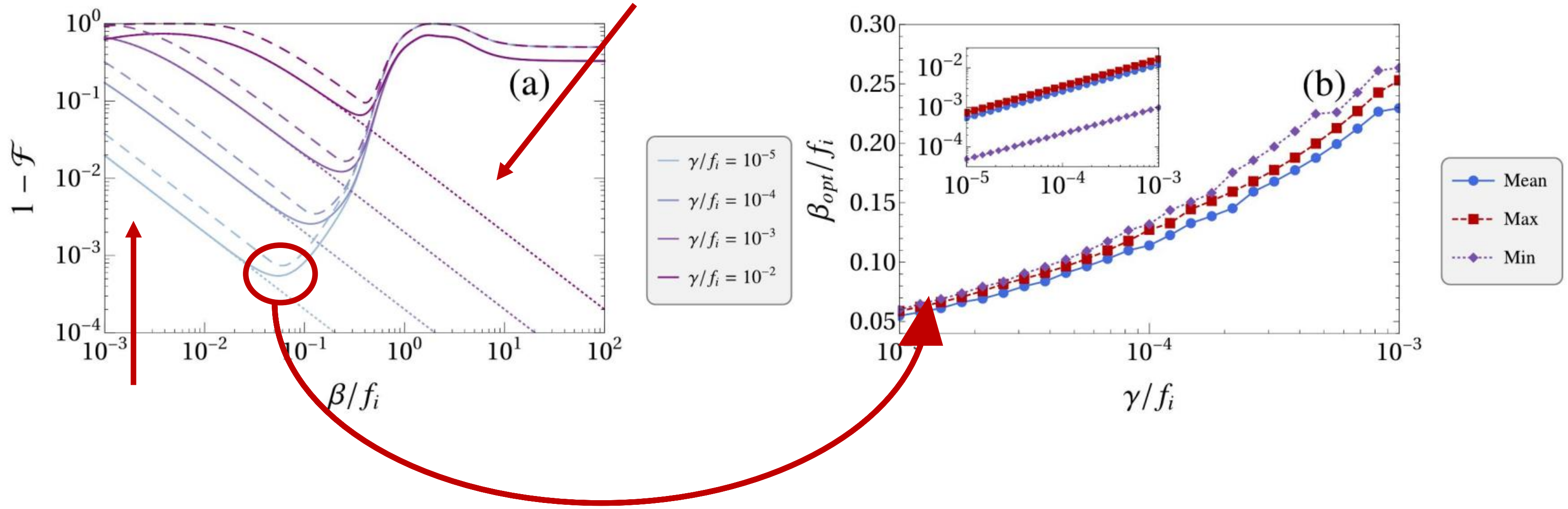
Simulations for the S gate. $f_{e0} = f_{e1} = f_i$.

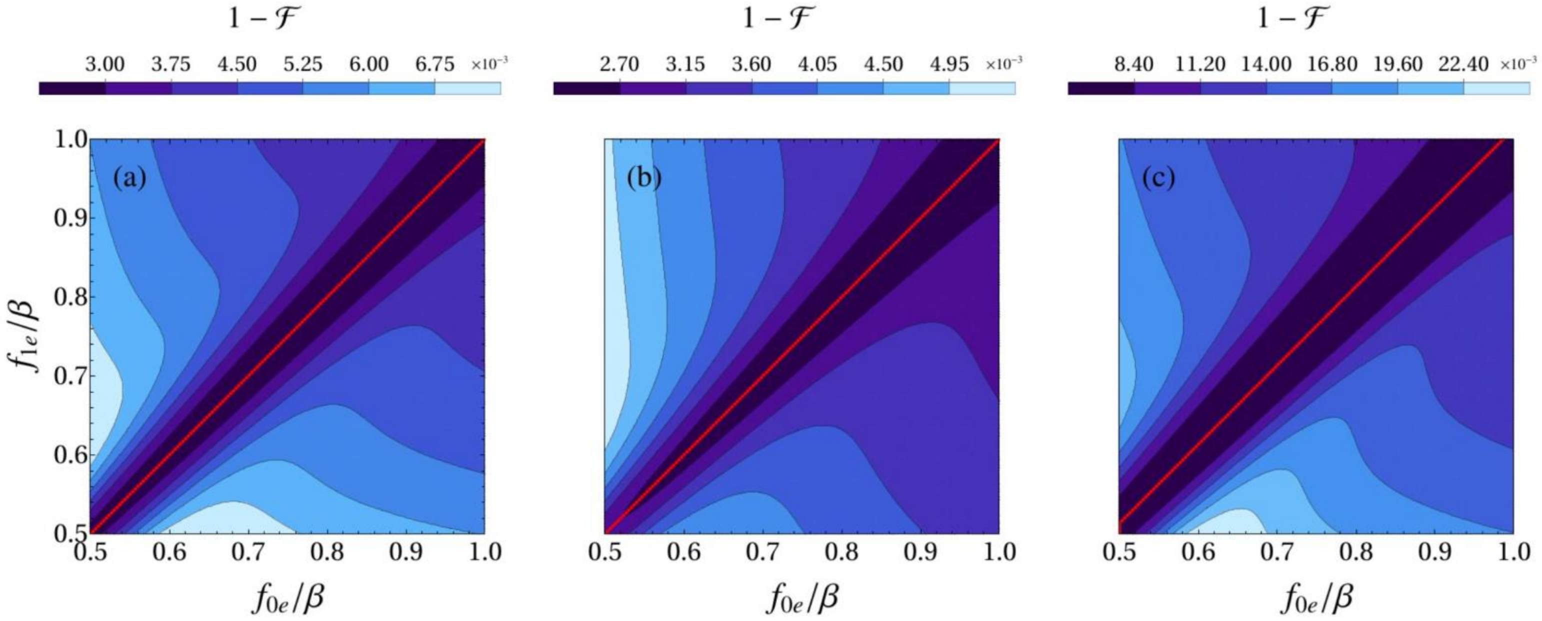


Heterogeneous frequencies

What happens when $f_{e0} \neq f_{e1}$?

Heterogeneous frequencies

What happens when $f_{e0} \neq f_{e1}$? Simulations for X, H and S:



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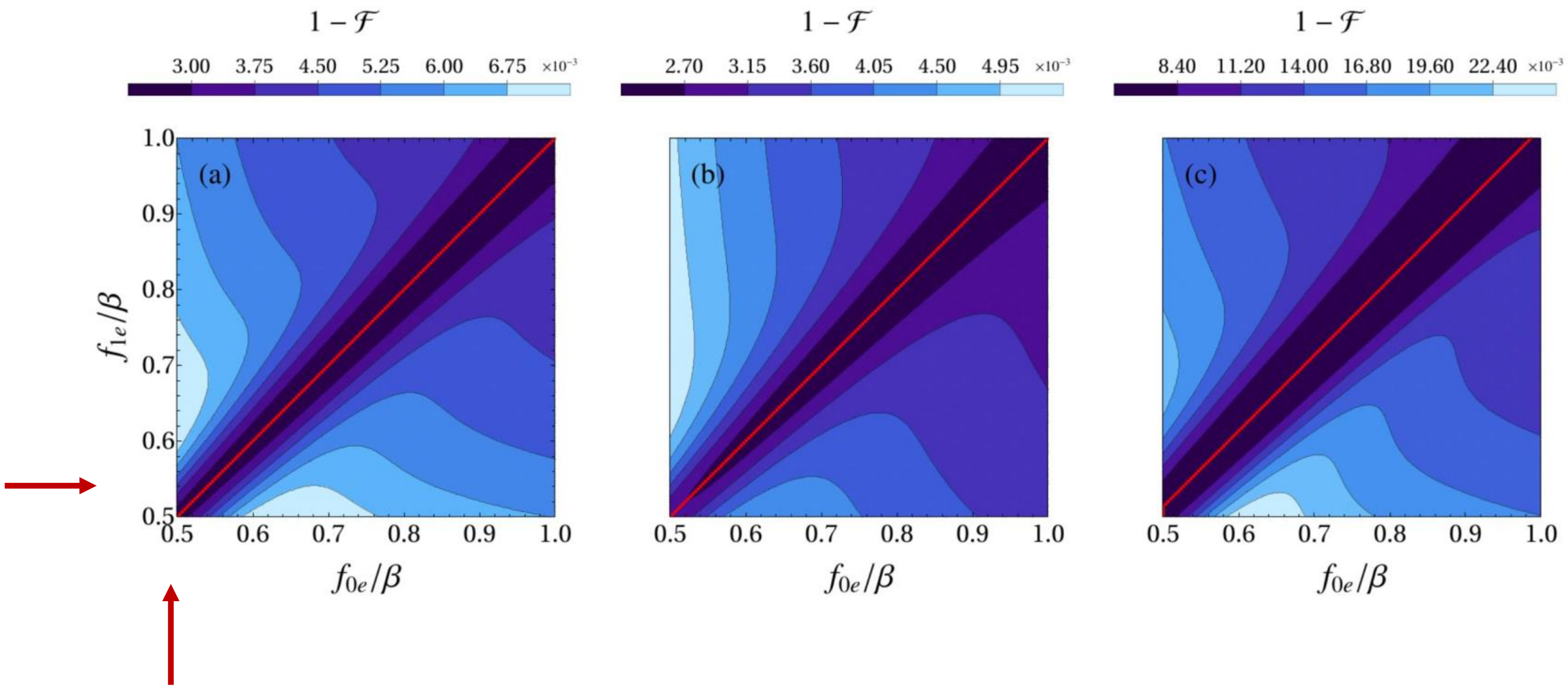

☞ X gate is symmetric ($\omega_0 = \omega_1 = 1/\sqrt{2}$). Other gates: $\omega_0 \neq \omega_1$

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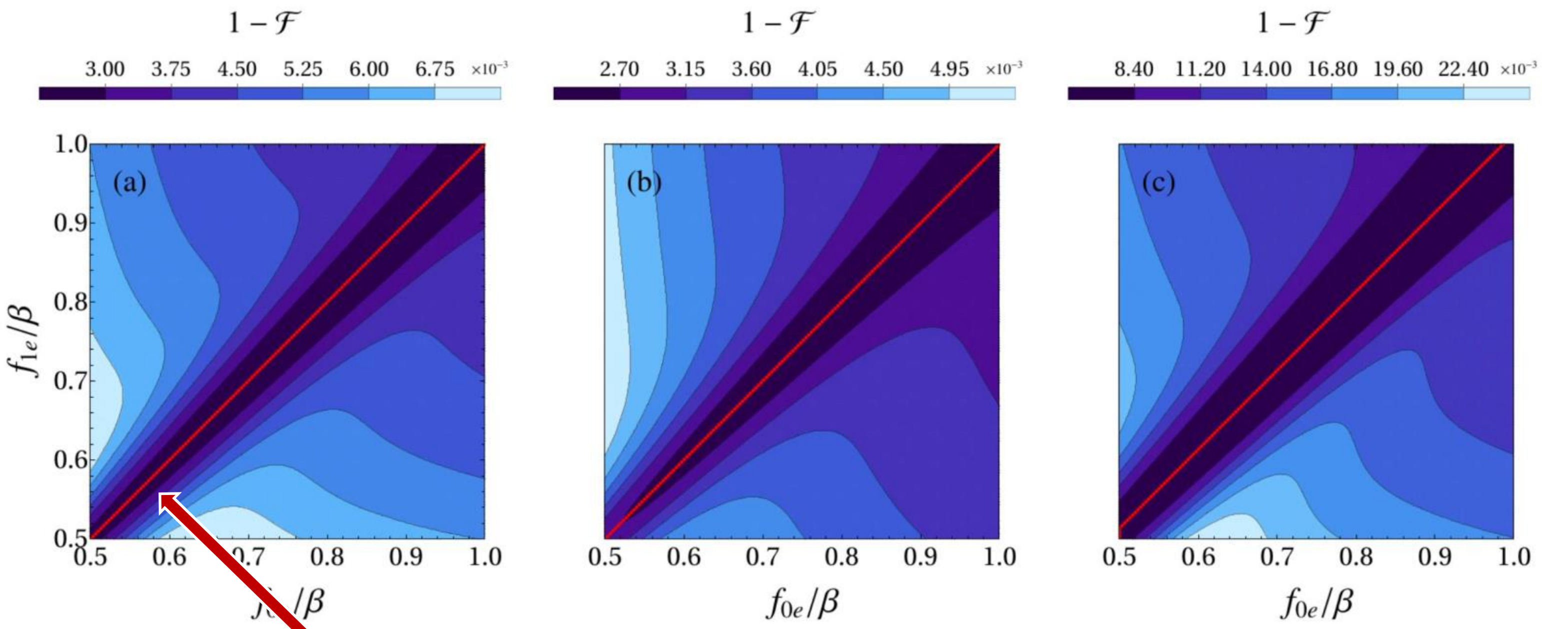

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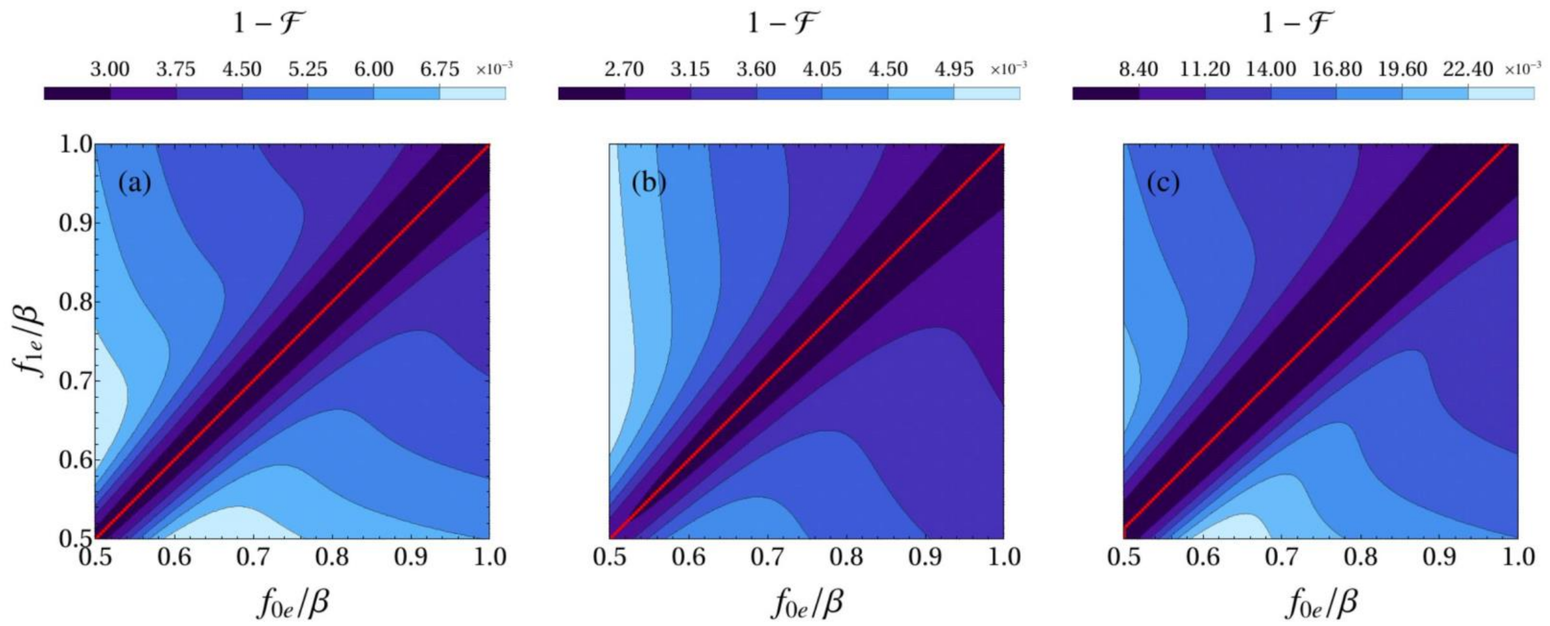

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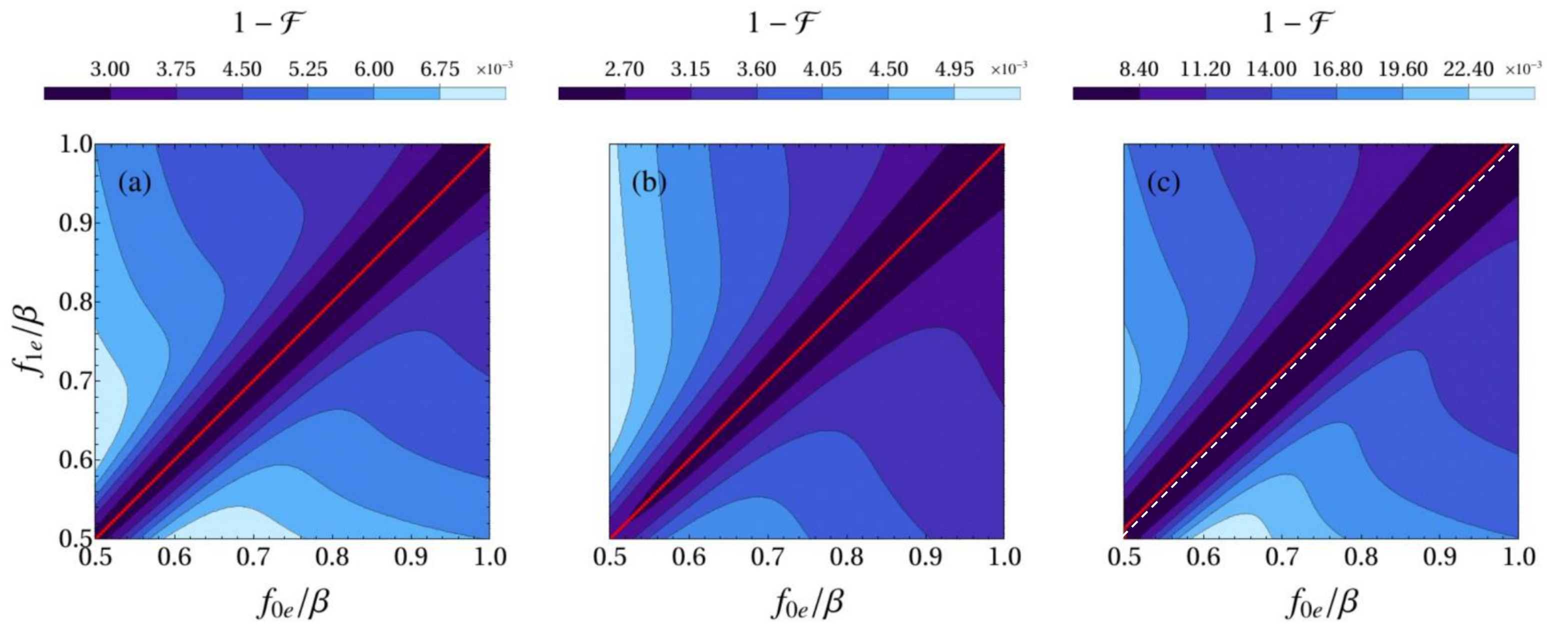

- X gate is symmetric ($\omega_0 = \omega_1 = 1/\sqrt{2}$). Other gates: $\omega_0 \neq \omega_1$
- Optimal strategy: increase the frequencies linearly
- $f_{e0} = f_{ej}$ for single pulse gates
- Slight offset for the S-gate

Heterogeneous frequencies

Hamiltonian in the dark-bright basis with counter-rotating terms:

$$H_{bd}(t) = \Omega(t)(1 + |\omega_0|^2 e^{-2if_{e0}t} + |\omega_1|^2 e^{-2if_{e1}t}) |e\rangle \langle b| \\ + \Omega(t)\omega_0\omega_1(e^{-2if_{e1}t} - e^{-2if_{e0}t})|e\rangle \langle d| + \text{h.c.}$$

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Dark state coupling!

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$f_{e0} = f_{e1}$ eliminates the coupling!

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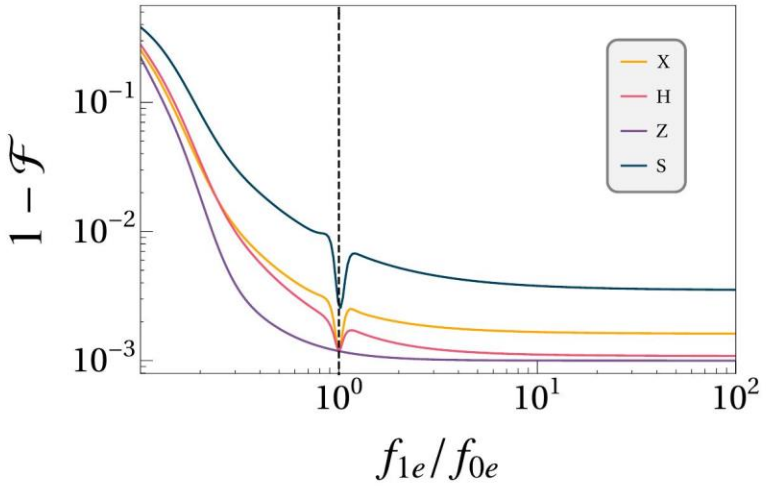

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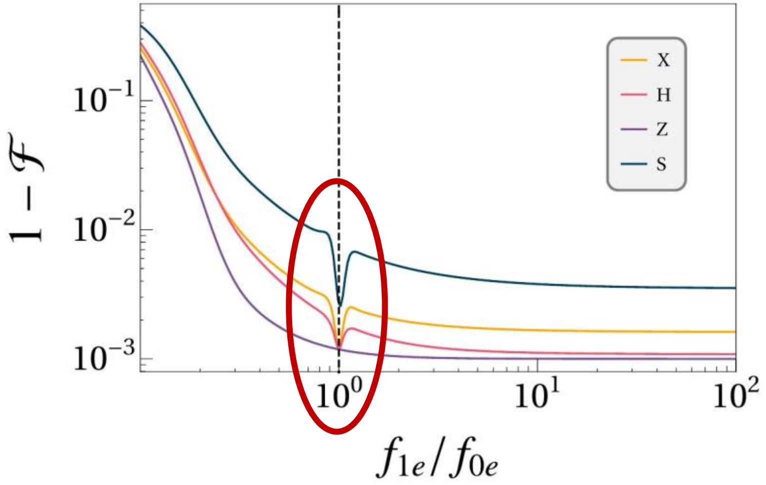
- ☞ Sudden improvement for $f_{0e} \approx f_{1e}$
- ☞ Heterogeneous frequencies $\rightarrow$ Relevance depends on the gate
- ☞ Experiment: $f_{1e}/f_{0e} \approx 1.04$ [a]



[a] Abdumalikov Jr et al., Nature **496**, 482–485 (2013).

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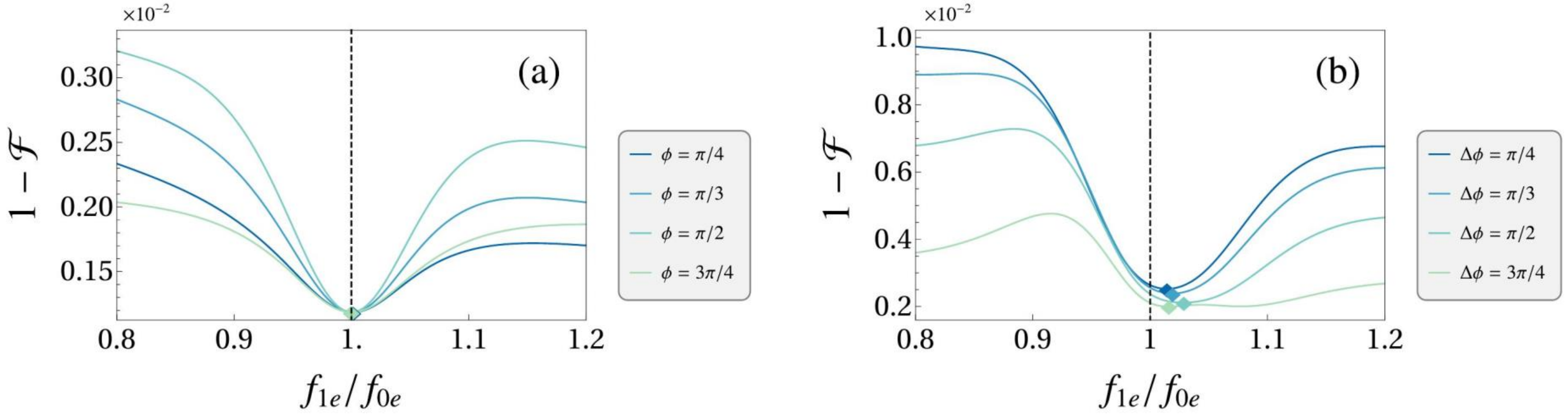

Conclusions

Conclusion and future outlooks

- The competing effects between dissipation and non-ideality of the RWA introduce a trade-off in the pulse length
- We obtained the optimal pulse length as a function of the system parameters numerically
- We have found that the presence of counter-rotating terms couple the dark state with the excited state
- This coupling in supressed in case of equal frequencies
- Keeping the two counter-rotating terms comparable results in a robust configuration
- Future outlooks: extension to other schemes, such as single loop configurations. Other pulse shapes?

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Thank you!

FAPESP IFUSP Instituto de Física da USP Erasmus+ UPPSALA UNIVERSITET

Appendix

Adiabatic evolution

Consider a vector of parameters $\mathbf{R}(t) = (R_1, R_2, ...)$ and a Hamiltonian:[2]

$$H(\mathbf{R}(t))\,|n(\mathbf{R}(t))\rangle = \epsilon_n(\mathbf{R}(t))\,|n(\mathbf{R}(t))\rangle$$

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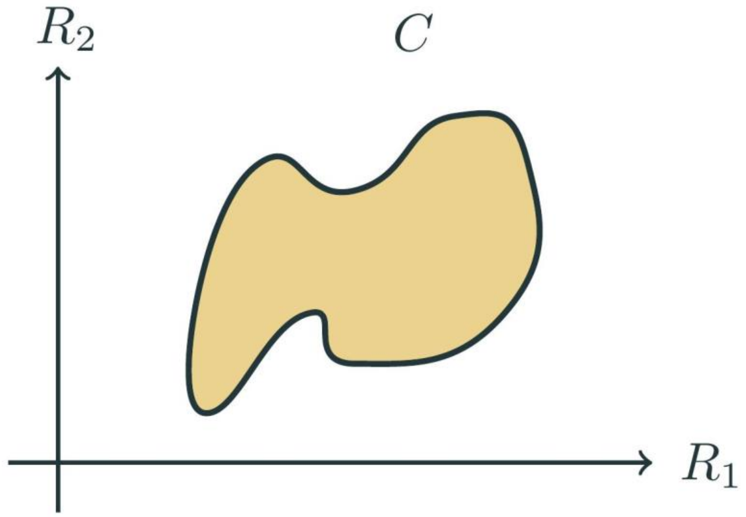

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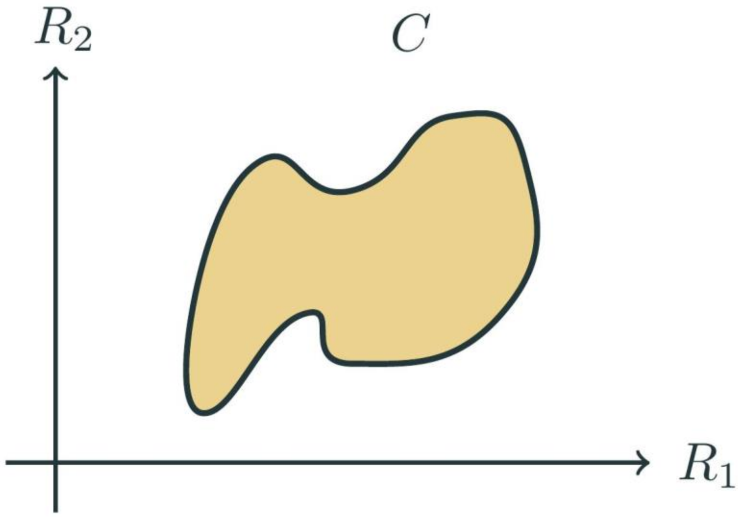

Arisal of a *geometric phase*:

$$\gamma_n(t) = \int_C \mathcal{A}^n(\mathbf{R}) \cdot d\mathbf{R}$$

where,

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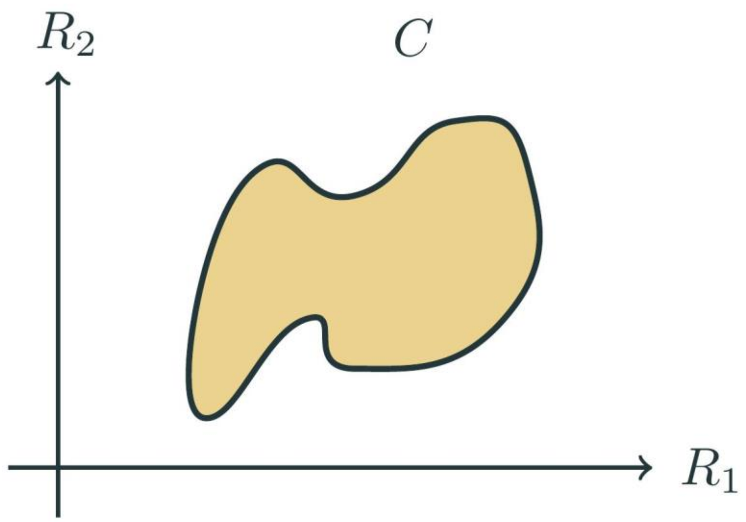

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Berry connection

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Parameter & eig. spc. structure

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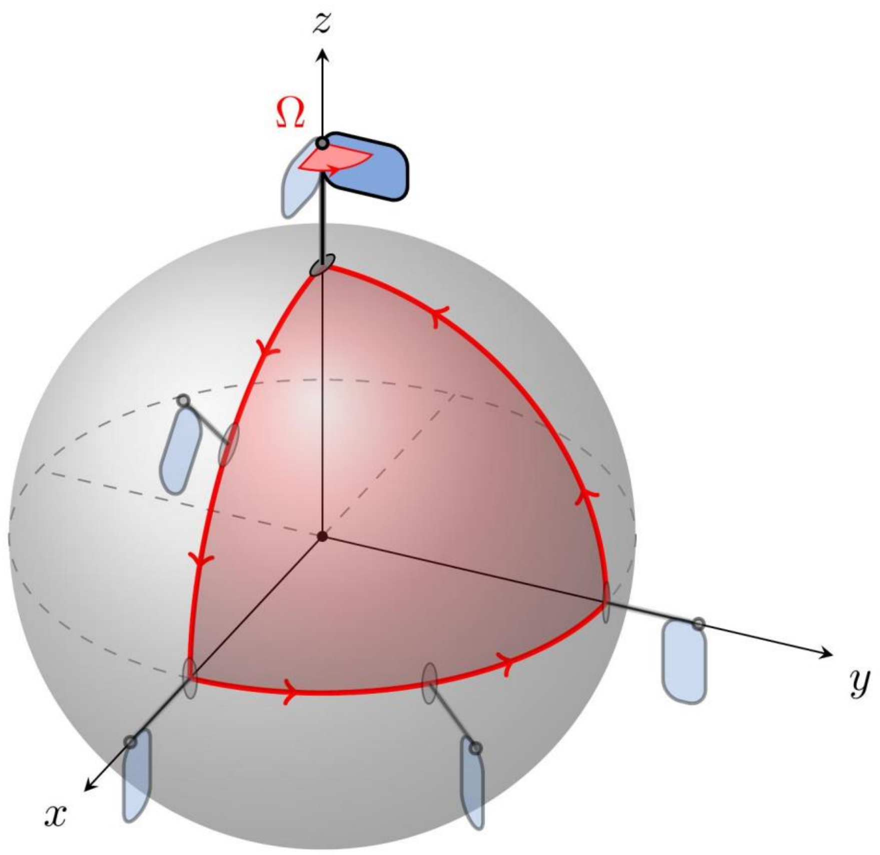
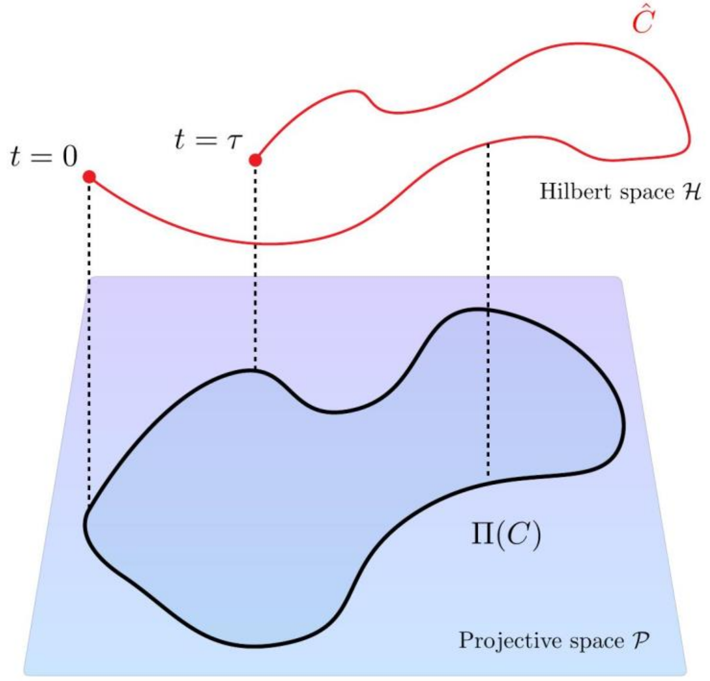
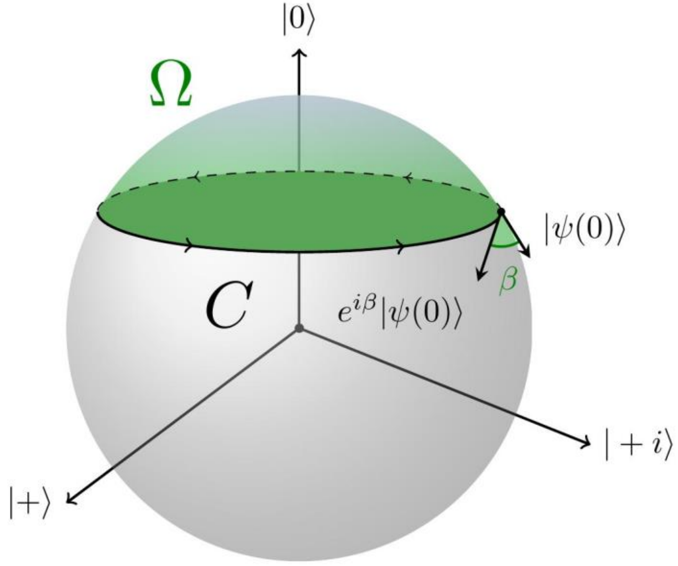
(Non)-Holonomy



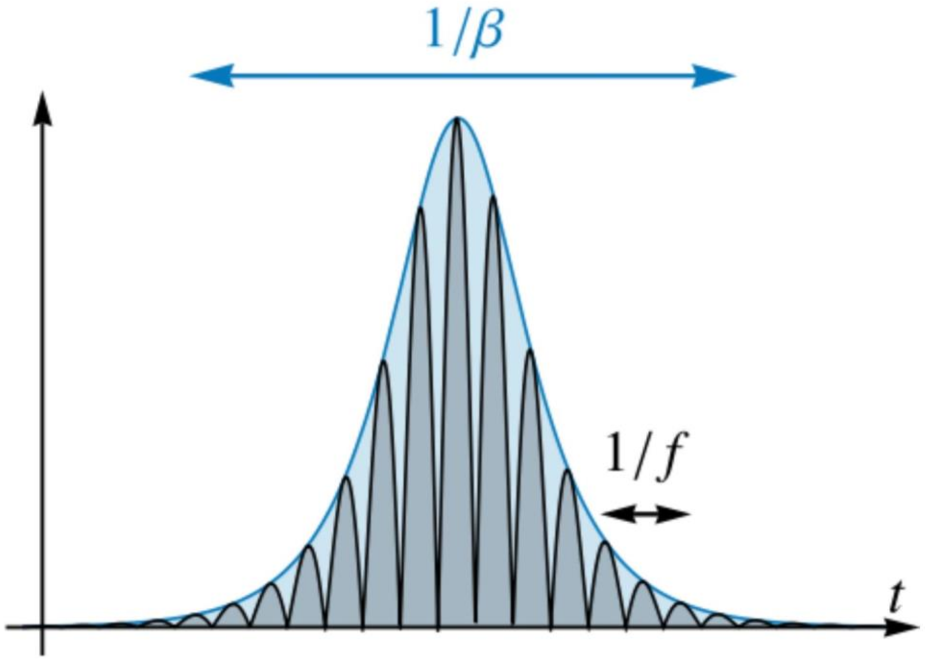
Aharonov-Anandan Phase





$$\beta = \int_0^\tau \langle \bar{\psi}(t) | \, i \frac{d}{dt} \, | \bar{\psi}(t) \rangle \, dt.$$

Validity of the RWA



Non-adiabatic non-Abelian geometrical phase

$$\boldsymbol{U}(t) = \mathcal{T}\exp\left\{\left(\int_0^t i(\boldsymbol{A} - \boldsymbol{K})dt\right)\right\}$$

☞ Dynamical: $K_{ab} := (1/\hbar)\langle\tilde{\psi}_a|H|\tilde{\psi}_b\rangle$

☞ Geometrical: $A_{ab} := i\langle\tilde{\psi}_a|d/dt|\tilde{\psi}_b\rangle$

Basis transformation $|\tilde{\psi}'\rangle = \boldsymbol{\Omega}|\tilde{\psi}\rangle$ implies:

$$\boldsymbol{A} \to i\boldsymbol{\Omega}^\dagger\dot{\boldsymbol{\Omega}} + \boldsymbol{\Omega}^\dagger\boldsymbol{A}\boldsymbol{\Omega}, \quad \boldsymbol{K} \to \boldsymbol{\Omega}^\dagger\boldsymbol{K}\boldsymbol{\Omega}$$

Aharonov-Anandan Phase